Conditionals

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1.1 Introduction

Conditional sentences are sentences of the form "if A, (then) C", as in the following examples:

(1) If this figure is a square, then it is a rectangle.

(2) If John comes to the party, Mary will be pleased.

(3) If John had come to the party, Mary would have been pleased.

In such sentences, the if-clause A is called the *antecedent* (sometimes *protasis*) of the conditional, and the then-clause C is called the *consequent* (or *apodosis*, viz. Carroll (1894)). Traditionally, conditional sentences have been taken to express hypothetical judgments, as opposed to categorical judgments (see the table of judgments in Kant (1781)), in that the speaker who expresses a sentence of the form "if A, C" does not assert C, but makes a weaker commitment, namely that C holds under the hypothesis expressed by A. For instance, in saying (1), the speaker expresses something weaker than if she had asserted "this figure is a rectangle", and similarly with (2) and (3).

The expression of conditionality in language does not necessarily involve explicit if-then constructions, as the following examples show:

(4) a. Kiss my dog and you'll get fleas (Bhatt and Pancheva (2006))
    b. If you kiss my dog, then you'll get fleas.

(5) a. No Hitler, no A-bomb. (Lewis (1973))
    b. If there had been no Hitler, there would have been no A-bomb.

(6) a. Unless you talk to Vito, you'll be in trouble.
    b. If you don't talk to Vito, you'll be in trouble.

However, all such sentences can be rephrased by means of an if-clause, as shown by the paraphrase given below them, and in this chapter we will focus on the semantic analysis of conditional sentences expressed with “if”.

From a typological point of view, at least three different kinds of conditional sentences are usually distinguished on semantic grounds, namely *indicative* conditionals, *counterfactual* conditionals (sometimes called *subjunctive* conditionals, although, as we will see below, the two notions are not exactly coextensional) and *relevance* conditionals (also known as biscuit conditionals, based on Austin’s example reproduced below). An illustration of the indicative-counterfactual distinction is given by (2) vs. (3) above. In (2), the speaker entertains as an open possibility that John comes to the party. In (3), a typical context for utterance is one in which the speaker takes it for granted that John did not come to the party. Thus, indicative and counterfactual conditionals differ primarily with regard to what is assumed about the *antecedent*.

In the case of relevance conditionals, somewhat symmetrically, the difference with indicative conditionals concerns primarily the attitude of the speaker toward the *consequent*. Some classic examples of relevance conditionals are the following:

(7) There are biscuits on the sideboard if you want them. (Austin (1961))

(8) If you’re looking for the captain, he isn’t here. (cited in Geis and Lycan (1993))

Unlike with ordinary indicative conditionals, consequents are asserted in such sentences. In Austin’s example, it is asserted that there are biscuits on the sideboard. Similarly, in Lycan and Geis’ example, it is asserted that the captain isn’t here. Thus, the role of the antecedent appears to be to make the information asserted in the consequent relevant for the purpose of the assertion itself. Relevance conditionals differ from indicative conditionals also in syntactic ways, in particular in that they typically block the insertion of “then” in front of the consequent:

(9) If you’re looking for the captain, *then he isn’t here.

This feature, to which we will return, can be seen as an indication that such conditionals do not express that the truth of the consequent depends in an essential way on the truth of the antecedent.

The semantic analysis of conditional sentences has been a hot topic since at least the Stoics (see Sanford (1989): chap. 1), and one will hardly find any other central construction in natural language for which so many semantic analyses still compete with each other. The goal of this chapter will be to present the most influential semantic frameworks to date, and the ways in which they can be used to cast light on the typology of conditionals into indicative, counterfactuals, and relevance conditionals. One important emphasis throughout the chapter will be the attention given to inferences involving the conditional in natural language, with a view to offering a systematic comparison between alternative frameworks. Another will concern the pragmatic-semantic distinction, and the question of whether all inferences that appear to be valid with the conditional can be captured by means of a uniform semantic mechanism.

The way this chapter is organized is as follows. Section 2 starts out with a review of the two-valued material analysis of conditionals, which we use a baseline for the presentation of competing analyses: as we will see, the material conditional captures several intuitive patterns of inference, but it both under- and over-generates with regard to very typical inferences we make in natural language. Sections 3 and 4 then focus on the main alternative to the material conditional analysis, namely the Stalnaker-Lewis analysis of conditionals as variably strict conditionals. Our presentation involves two steps: section 3 compares Stalnaker's and Lewis's respective theories, while section 4 points out some problematic predictions they make regarding the relation between conditionals, conjunctions and disjunctions. Sections 5 and 6 are devoted to refinements of the Lewis-Stalnaker analysis with an eye to the syntax-semantics interface. In section 5, in particular, we compare a referential elaboration on the Lewis-Stalnaker's semantics in terms of plural definite descriptions with a quantificational analysis in which conditionals are treated as strict conditionals with a mechanism of variable domain restriction. Section 6 considers another such refinement, namely the view of if-clauses as restrictors of generalized quantifiers due to Lewis and Kratzer. From a semantic point of view, this view too essentially implements the truth conditions proposed by Stalnaker-Lewis, but again it makes specific predictions about the syntax-semantics interface, in particular regarding the embedding of conditionals under the scope of various operators.

What distinguishes the Stalnaker-Lewis analysis and its variants from the material conditional analysis is that the former are not truth-functional, unlike the latter. In section 7, we turn to another family of alternatives to the Boolean analysis, one that maintains truth-functionality but introduces defective truth conditions for conditionals in a three-valued setting. For indicative conditionals at least, this approach appears quite natural, though arguably less so for counterfactuals. More generally, indicative conditionals are our main focus throughout most of sections 3 to 7, in particular because each of the various semantics we consider can be motivated in relation to specific inferences licensed or not by the indicative conditional (Lewis's semantics is an exception, but can actually be applied to indicative conditionals proper). In sections 8 and 9, we move away from the examination of semantic frameworks to focus on the specificity of counterfactual conditionals on the one hand (section 8) and relevance conditionals on the other (section 9). The Appendix, finally, gives a quick comparison between the main frameworks.

1.2 The material conditional

The most venerable analysis of conditional sentences dates back to Philo of Megara (fl. 300 BCE) and was later taken up by Frege and by Russell at the inception of modern logic (Frege (1879), Russell (1903)). On this approach, conditional sentences of the form "if *A*, *C*" are handled truth-functionally, which means that the truth value of a conditional sentence is a Boolean function of the truth values of the antecedent and consequent. According to Philo, "the conditional is true when it does not start with the true to end with the false; therefore, there are for this conditional three ways of being true, and one of being false" (Sextus (n.d.) VIII, 417). In modern terms, a sentence with a material conditional will be represented as $A \supset C$, and what is assumed is that $[\![A \supset C]\!] = 0$ provided $[\![\mathrm{A}]\!] = 1$ and $[\![\mathrm{C}]\!] = 0$. Because the logic is assumed to be bivalent, this means that $[\![A \supset C]\!] = 1$ if and only if $[\![\mathrm{A}]\!] = 0$ or $[\![\mathrm{C}]\!] = 1$, or equivalently, provided $[\![\mathrm{A}]\!] \leq [\![\mathrm{C}]\!]$. That is, a material conditional is true if and only if either its antecedent is false, or its consequent is true. As can be checked, the material conditional can be integrally defined in terms of negation and conjunction, as $\neg(A \wedge \neg C)$, which captures exactly the constraint that the conditional is true unless the antecedent is true and the consequent false.

In order to see whether the material conditional analysis adequately captures our truth conditional intuitions about conditional sentences, let us first consider some patterns of inference that are supported by this analysis. As is standard, we will say that a set of sentences Γ entails a sentence ψ (noted $\Gamma \models \psi$) iff every model that makes all sentences of Γ true makes ψ true. Some valid schemata of inference supported by the material conditional are the following:

(10) a. $A \supset C, A \models C$ (modus ponens)
b. $A \supset C, \neg C \models \neg A$ (modus tollens)
c. $(A \vee C) \models (\neg A \supset C)$ (or-to-if)[1]
d. $(A \wedge B) \supset C \equiv (A \supset (B \supset C))$ (import-export)

[1] This inference is also called the Direct Argument in Stalnaker (1975).

e. $(A \vee B) \supset C \equiv (A \supset C) \wedge (B \supset C)$ (simplification of disjunctive antecedents)

Those schemata are worth singling out, because they are generally considered to be intuitively acceptable for both indicative conditionals and counterfactual conditionals. Other valid schemata according to the material conditional analysis are considered to be more problematic, in particular:

(11)
a. $\neg A \models A \supset C$ (falsity of the antecedent)
b. $C \models (A \supset C)$ (truth of the consequent)
c. $A \supset C \models \neg C \supset \neg A$ (contraposition)
d. $A \supset C \models A \wedge B \supset C$ (strengthening of the antecedent)
e. $A \supset B, B \supset C \models A \supset C$ (transitivity)

Doubts about the first two patterns of inference were raised early on by C.I. Lewis (Lewis (1912)), one of the founders of modal logic, on the grounds that under the material conditional analysis, any true sentence is thus entailed by any other, and conversely any false sentence entails any other. Because of that, schemata (11)-a and (11)-b are known as the 'paradoxes of material implication'. Some putative examples of the oddity of those schemata might be:

(12) Paris is the capital of France. ?? Therefore if Paris is not the capital of France, Obama is a Republican.

(13) John was in London this morning. ?? So if John was in Paris this morning, John was in London this morning.

Doubts about the other patterns of inference were raised a bit later by Goodman (1947) in particular,[2] and largely motivated the analysis of conditionals later proposed by Stalnaker and Lewis. Some examples of infelicitous inferences based on those schemata are:

(14) If Goethe had lived past 1832, he would not be alive today. ??So, if Goethe were alive today, he would not have lived past 1832. (Kratzer (1979))

(15) If this match is struck, it will light. ?? So if this match is soaked overnight and it is struck, it will light. (after Goodman (1947))

(16) If I quit my job, I won't be able to afford my apartment. But if I win a million, I will quit my job. ?? So if I win a million, I won't be able to afford my apartment. (Kaufmann (2005))

[2] The case of transitivity appears to be first discussed by Stalnaker (1968).

Another often pointed out inadequacy of the material conditional analysis with regard to natural language concerns the interplay of the conditional with negation. In classical two-valued logic, we have that:

(17) $\neg(A \supset C) \equiv A \wedge \neg C$ (conditional negation)

With indicative conditionals, one often understands "if A, not C" to imply "not (if A, C)" and conversely (viz. Carroll (1894) for a symptom of this problem). However, classically, although $\neg(A \supset C) \models A \supset \neg C$, it is not the case that $A \supset \neg C \models \neg(A \supset C)$. Moreover, the inference from $\neg(A \supset C)$ to $A \wedge \neg C$ appears too strong. Under the material conditional analysis, one would predict:

(18) It is not the case that if God exists, criminals will go to heaven. ?? So, God exists, and criminals will not go to heaven. (attributed to Anderson and Stevenson, cited in Lycan (2001))

Whether these inferences are inadequate on semantic or on pragmatic grounds has been and remains an issue. (For instance, consider the conditional (1) above. Clearly, contraposition seems to be a sound rule in that case, why is it sound here, and unsound there?). The answer to this question also depends on the prospects for having a unified analysis of indicative and counterfactual conditionals. Quine (1950), for instance, essentially considered the paradoxes of material implication, and the other problematic inferences as pragmatic anomalies. The same attitude is taken by Grice (1989) and Lewis (1973) on indicative conditionals. Grice (1989), in particular, entertains the idea that an application of the maxim of Quantity ("*make your contribution as informative as required (for the current purposes of the exchange)*") might handle some of the difficulties of the material conditional. This strategy, arguably, might explain why it would be awkward to infer $A \supset C$ from C (since $A \supset C$ is less informative than C). But it does not straightforwardly account, for example, for why the negation of a conditional is so often understood as a conditional negation (a problem Grice regards as "a serious difficulty" for his account). As pointed out, under the material analysis of the conditional, $\neg(A \supset C)$ entails $A \supset \neg C$, but the converse is not true. Hence, this would be a case in which what is inferred is logically weaker (and so less informative) than what is literally asserted.

Irrespective of whether the material conditional can give a good analysis of indicative conditionals, Quine considered the material conditional analysis to be semantically inadequate for counterfactuals. His point was that:

"Whatever the proper analysis of the contrafactual conditional may be, we may be sure in advance that it cannot be truth-functional; for, obviously ordinary usage demands that some contrafactual conditionals with false antecedents and false consequents be true and that other contrafactual conditionals with false antecedents and false consequents be false" (Quine (1950)).

An example of such a pair for Quine is:

(19) a. If I weighed more than 150 kg, I would weigh more than 100 kg.
b. If I weighed more than 150 kg, I would weigh less than 25 kg.

Suppose the speaker weighs exactly 70kg. Then, both antecedents and consequents can be taken to be false (putting all component sentences in the present tense), yet the first counterfactual is intuitively true, and the second false. Interestingly, putting the two sentences both in present tense actually suggests that (*pace* Quine) the material conditional is equally inadequate to deal with indicative conditionals, since a sentence like "if I weigh more than 150kg, then I weigh more than 100kg" is intuitively true irrespective of whether one weighs 70 kgs or not, whereas "if I weigh more than 150kg, I weigh less than 25kg" is intuitively false irrespective of the speaker's weight again.

In summary, we see that while the material conditional certainly captures some of the conditions under which an indicative conditional sentence is judged false, it supports some inferences whose validity is problematic in relation to both indicative and counterfactual conditionals. Furthermore, a two-valued truth-functional analysis simply fails to account for cases in which a conditional with a false antecedent is not automatically judged true. Yet further arguments hold against the material conditional, in particular the fact that it predicts inadequate truth conditions for if-clauses under the scope of specific operators (see below section 1.6).

1.3 Strict and variably strict conditionals

The first attempt to fix the inadequacies of the material conditional was made by C. I. Lewis (viz. Lewis (1918)) with the definition of the strict conditional, intended to block the paradoxes of material implication, and going beyond the truth-functional analysis. A strict conditional is a material conditional under the scope of a necessity operator. On that analysis, "if A, C", means "necessarily, if A, C", which we will represent as $\Box(A \supset C)$.[3] Treating necessity oper-

[3] See G.E. and Cresswell (1996) for a presentation of modal logic and strict conditionals. Frege (1879) should be given credit for an anticipation of the strict conditional view. Frege distinguishes explicitly the material conditional $A \supset C$ from the quantified conditional $\forall x(A(x) \supset C(x))$ relative to natural language. In § 5 of Frege (1879), he writes about the material conditional: "the causal connection inherent in the word "if", however, is not

ators as universal quantifiers over possible worlds, this means that the logical form of conditional sentences is as follows:

(20) $\forall w(A(w) \supset C(w))$

It is easy to see that this analysis blocks the paradoxes of material implication. For example, the counterpart of the schema of truth of the consequent, $C \models A \supset C$ now is: $C \models \Box(A \supset C)$. The schema would be valid provided the following entailment held in first-order logic:

(21) $C(i) \models \forall w(A(w) \supset C(w))$

However, C may hold at world i without holding at all worlds in which A holds. Similarly, a strict conditional analysis can account for Quine's pair, namely for why we judge (19-a) true and (19-b) false, even assuming the antecedent to be false at the actual world. Despite this, it is easy to see that the schemata of contraposition, strengthening of the antecedent and transitivity all remain valid under the strict conditional analysis. This is due to the fact that, seen as a universal quantifier, the strict conditional is downward monotone on its antecedent, and upward monotone on its consequent.

A direct relative of the strict conditional that fixes that problem is the conditional of Stalnaker-Lewis (Stalnaker (1968), Lewis (1973)), which Lewis has dubbed a 'variably strict conditional', essentially because, where a strict conditional says: 'if A, C' is true provided C is true in all the worlds where A is true, the variably strict conditional says: 'if A, C' is true provided C is true in all the *closest* worlds to the actual world where A is true, where closeness depends on the world of evaluation.

The initial motivation for Stalnaker's analysis of "if A, C", based on an insight originally due to Ramsey (1929), is presented by him as follows:

> "first, add the antecedent hypothetically to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true." (Stalnaker (1968)).

In Stalnaker's possible world framework, the notion of minimal adjustment is described in terms of selection functions. Given a pair consisting of a world w and antecedent A, $f(A,w)$ is taken to denote the closest world w' to w that makes the antecedent A true. Given this apparatus, a conditional "if A, C" is true at world w if and only if C is true at the closest A-world to w.

expressed by our signs"; in §11, he then writes about the quantified form: "*this is the way in which causal connections are expressed*" (his emphasis).

More formally, let '>' stand for the conditional operator, and define a selection model to be a structure $\langle W, R, V, \lambda, f \rangle$, where W is a set of worlds, R is a reflexive accessibility relation on W, V a valuation of the atomic sentences on worlds in W, λ is the so-called absurd world (satisfying every sentence), and f is a selection function (from pairs of propositions and worlds to worlds). Given such a model M, the truth conditions for conditional sentences of the form $A > C$ are as follows:

(22) $M, w \models A > C$ iff $M, f(A, w) \models C$ (Stalnaker's semantics)

Selection functions satisfy five conditions, namely:

(23) a. $f(A, w) \models A$
b. $f(A, w) = \lambda$ only if there is no w' such that wRw' and $w' \models A$
c. if $w \models A$, then $f(A, w) = w$
d. if $f(A, w) \models C$ and $f(C, w) \models A$, then $f(A, w) = f(C, w)$
e. if $f(A, w) \neq \lambda$, then $wRf(A, w)$

Clauses (23)-b and (23)-e ensure that the selected world is the absurd world exactly when no possible world satisfies the antecedent.[4] Clause (23)-a means that the closest A-world is an A-world, and (23)-c that the closest A world is the actual world if the actual world satisfies A (a proviso also called Centering). Clause (23)-d, finally, is needed to ensure consistency in the ordering of possible worlds induced by the selection function (whereby if $f(A, w) = w'$, then w' is prior to all other worlds in which A is true).

Like the strict conditional analysis, Stalnaker's semantics invalidates the paradoxes of material implication, but this time it also invalidates contraposition, strengthening of the antecedent, and transitivity. For instance, consider the problematic instance of strengthening of the antecedent in (15). Assume that the closest world in which John adds sugar in his coffee is a world in which he finds it better ($f(sugar, w) \models better$). This is compatible with the idea that the closest world in which he adds sugar and salt is a world in which he does not find it better ($f(sugar \wedge salt, w) \models \neg better$). This implies that the closest world in which John adds sugar is not a world in which he also adds salt with it.

Unlike Stalnaker, Lewis does not use selection functions, however his semantics for counterfactuals involves the notion of similarity or closeness between worlds. The main difference with an approach in terms of selection functions is that Lewis drops two requirements underlying Stalnaker's approach,

[4] (23)-e is not originally included in Stalnaker (1968) but is needed to ensure the converse of (23)-b, see Nute (1980).

the so-called *Uniqueness assumption*, and the *Limit assumption*. The Uniqueness assumption is the assumption that for every antecedent A and world w, there is at most one closest A-worlds to w. The Limit assumption is the assumption that for every antecedent A and world w, there is at least one closest A-worlds to w. In order to capture those differences, Lewis's models fix an explicit similarity relation between worlds, where $x \leq_w y$ means that x is closer to w than y.[5] Lewis's truth conditions for conditionals are the following: "if A, C" is true in w" iff either A holds in no possible world, or every world where A and C are true together is more similar to w than any worlds where A and $\neg C$ hold together, that is:[6]

(24) $M, w \models A > C$ iff either there is no w' such that wRw' and $w' \models A$, or there is an x such that $M, x \models A \wedge C$ such that there is no y such that $y \leq_w x$ and $M, y \models A \wedge \neg C$ (Lewis's semantics)

Like Stalnaker's semantics, Lewis's semantics invalidates contraposition, strengthening of the antecedent, and transitivity. It makes three predictions regarding natural language that depart from Stalnaker's system, however.

The first relates to the Uniqueness assumption, and concerns the schema of conditional excluded middle (CEM), that is:

(25) $(A > C) \vee (A > \neg C)$ (Conditional Excluded Middle)

CEM is valid on Stalnaker's semantics, because the closest A-world is necessarily a C world or a non-C world. Lewis's semantics however permits ties between worlds. In particular, there can be two A-worlds that are equally close to the actual world, such that one is a C-world, and the other is a non-C world. An example in support of this prediction is Quine's Bizet-Verdi example, where neither counterfactual seems to be true:

(26) a. If Bizet and Verdi had been compatriots, they would have been French.
b. If Bizet and Verdi had been compatriots, they would have been Italian.

A second and related difference concerns the treatment of negation. In Stalnaker's semantics, provided A is a possible antecedent, the negation of a condi-

5 The relation $\leq_w$ is a weak ordering, namely it is transitive, and complete. Further assumptions are made by Lewis concerning the actual world, and the relation between accessible and inaccessible worlds from a given world.

6 Note that Lewis's symbol for the conditional is $\Box\!\!\rightarrow$. We deliberately use the same symbol $>$ for both Lewis's and Stalnaker's conditional connective.

tional $\neg(A > C)$ is equivalent to the conditional negation $(A > \neg C)$. This is an important difference with the material conditional analysis. As the Bizet-Verdi example shows, however, in Lewis's approach, $\neg(Compatriots > Italians)$ does not imply $(Compatriots > \neg Italians)$.

The third difference finally pertains to the Limit assumption. Lewis points out that if we say things like "if this line were longer than it is,...", about a line one inch long, there is no closest world where the line is more than one inch long, at least if we order worlds with regard to how little the line differs in size from its actual size. Lewis's argument is cogent. However, it also creates problems. An objection made by Stalnaker (1980) against giving up the Limit assumption concerns a similar case. Suppose Mary is 5cm shorter than Albert. Clearly, according to Lewis's metric of similarity, there will be closer and closer worlds where Mary is taller than she actually is. In that case, Lewis's semantics predicts the truth of:

(27) If Mary were taller than she is, she would be shorter than Albert.

The reason is that a world were Mary is 1cm taller than she is is a world where she is taller than she actually is, and closer to the actual world than any world in which she is taller than she is and taller than Albert. However, it seems one would like to say:

(28) If Mary were taller than she is, she might be taller than Albert

In Lewis's semantics, 'might' counterfactuals are defined as duals of 'would' counterfactuals, so as of the form $\neg(A > \neg C)$. If (27) is true, then (28) must be false, which is inadequate. There is a way out for Lewis, however, namely to assume a different metric of similarity, and for instance to postulate that all worlds in which Mary is up to 5cm taller than she is are equally close to the actual world. As emphasized by Schlenker (2004), however, this move amounts to restoring the Limit assumption.

With regard to the set of valid formulae, Conditional Excluded Middle is the distinguishing principle between Lewis's and Stalnaker's system. As it turns out, failure of Uniqueness or of the Limit assumption suffice to invalidate CEM. However, both the Limit assuption and the Uniqueness assumption are required to make CEM a valid principle. Interestingly, it can be seen that CEM is not a valid principle either under the analysis of conditionals as strict conditionals. In both Lewis's and Stalnaker's semantics, however, the conditional is intermediate between a strict and a material conditional, that is we have (assuming a reflexive relation of accessibility for the necessity operator):

(29) $\models \Box(A \supset C) \supset (A > C) \supset (A \supset C)$

Thus, the Stalnaker-Lewis conditional is weaker than a strict conditional (in that it is non monotonic), but like the latter it remains stronger than the material conditional.

1.4 *If*, *and* and *or*

We saw that the conditional of Stalnaker-Lewis invalidates some problematic laws of the material conditional. At the same time, it fails to validate three laws that we initially listed as plausible for indicative and counterfactual conditionals, namely the law of import-export (IE), the law of or-to-if (OI) and the law of simplification of disjunctive antecedents (SDA):

(30) a. $(A > (B > C)) \models (A \wedge B > C)$ (IE)
b. $(A \vee C) \models (\neg A > C)$ (OI)
c. $(A \vee B) > C \models (A > C) \wedge (B > C)$ (SDA)

IE fails since the closest B-world(s) to the closest A-world(s) need not be the closest $A \wedge B$-worlds. Similarly, SDA fails because the closest $A \vee B$-world(s) may only satisfy B and not A, and so the closest A-world(s) may fail to make C true. Finally, OI fails because $A \vee C$ may be true only because A is true: thus, the closest $\neg A$ world(s) may very well fail to satisfy C.

Each of these inferences is generally considered to be highly plausible, which suggests that some amendment is needed on the semantics proposed by Stalnaker and Lewis: either by the consideration of some additional semantic mechanism, or by some deeper modification of the semantics itself. Let us consider IE first, which has led McGee to propose a revision of the semantics of Stalnaker and Lewis.[7] According to McGee, the validity of IE is “a fact of English usage, confirmed by numerous examples” (McGee (1989):489). McGee’s semantics accommodates IE essentially by the following modification of Stalnaker’s semantics: instead of defining truth relative to a world only, it defines truth relative to a world and a factual (Boolean) hypothesis. Assuming $f(A, w) \neq \lambda$, the semantics goes as follows, for A and B being factual sentences:

[7] McGee’s motivations are actually deeper, as McGee’s proposal is to establish a link between Stalnaker’s possible world semantics and Adams’ probabilistic semantics for conditionals (Adams (1975)), on which an argument is probabilistically valid iff the premises cannot be highly probable without the conclusion being highly probable. See Adams (1998) for a comprehensive account of the notion of probabilistic validity, and our remarks below in section 1.7.

(31) a. $M, w \models_A p$ iff $M, f(A, w) \models p$
b. $M, w \models_A \neg\phi$ iff $M, w \not\models_A \phi$
c. $M, w \models_A (\phi \wedge \psi)$ iff $M, w \models_A \phi$ and $M, w \models_A \psi$.
d. $M, w \models_A (\phi \vee \psi)$ iff $M, w \models_A \phi$ or $M, w \models_A \psi$.
e. $M, w \models_A (B > \phi)$ iff $M, w \models_{(A \wedge B)} \phi$

By definition, $M, w \models \phi$ iff $M, w \models_\top \phi$. Clause (31)-e is what ensures the validity of IE. The validity of IE, on the other hand, forces some other prima facie plausible features of the Stalnaker-Lewis conditional to drop.[8] The most spectacular case concerns the schema of *modus ponens*. In McGee's framework, *modus ponens* remains valid for unembedded conditionals, but can fail for compound conditionals.[9] An example given by McGee (1985) for this failure is the following:

(32) a. If a Republican wins the election, then if it's not Reagan who wins the election, it will be Anderson.
b. A Republican will win the election.
c. If it's not Reagan who wins the election, it will be Anderson.

The context of McGee's example is that of the 1980 US presidential elections, in which Reagan was ahead of the polls, with Carter as the main Democrat contender, and Anderson as a "distant third". In this context, McGee points out that one can accept (32)-a and (32)-b and refuse to accept (32)-c, based on the belief that if it's not Reagan who wins the election, it will be Carter. Let M be a model in which at w: $M, w \models_{(R \vee A) \wedge \neg R} A$, $M, w \models R \vee A$ and $M, w \not\models_{\neg R} A$. The first premise says that the closest world where either Reagan or Anderson is elected, and where Reagan is not elected, is a world where Anderson is elected. The conclusion however denies that the closest world where Reagan is not elected is a world where Anderson is elected. Importantly, this can be true only because the closest world where Reagan does not win the election is no longer assumed to be a world where a Republican wins.

It should be noted that in McGee's semantics, the sentential schema $(R \vee A) > (\neg R > A))$ (usable to get a logical paraphrase of (32)-a), is valid. In argument form, however, the inference from Or to If is not valid, just as in Stalnaker's and Lewis's case.[10] In Stalnaker's or Lewis's semantics, indeed, this can fail to hold if the closest world where Reagan or Anderson wins is a world where

[8] Relatedly, McGee appears to give up the postulate that $f(A, w) = w$ whenever $w \models A$.
[9] See also Yalcin (2012) for a recent attack on *modus tollens* for conditionals. Yalcin's proposed counterexample involves the interaction of conditionals with the modal "probably".
[10] This exemplifies the failure in McGee's framework of the deduction theorem, namely of the equivalence beween $A \models C$ and $\models A > C$.

only Reagan wins. According to Stalnaker (1975), the failure of Or-to-If is not a defect of the semantics, for if it were valid, from the fact that $A \models A \vee B$, one could infer $A \models \neg A > B$, a form of the paradox of the falsity of the antecedent. Stalnaker's suggested treatment for Or-to-If is pragmatic because of that. On Stalnaker's view, OI is indeed a *reasonable* inference, though not a valid inference. The way Stalnaker captures the notion of a reasonable inference is by introducing the notion of the context set, that is the set of worlds compatible with what the speaker presupposes. According to him "when a speaker says "if A", then everything he is presupposing to hold in the actual situation is presupposed to hold in the hypothetical situation in which A is true". This is reflected in Stalnaker's system by a defeasible constraint on selection functions, which is:

(33) **Constraint on selection**: the world selected by the antecedent of a conditional must be a world of the context set

An additional assumption needed by Stalnaker is that in order to utter "*A* or *B*", the speaker must allow each disjunct to be true without the other in the context set (the context set contains $A \wedge \neg B$-worlds and $\neg A \wedge B$-worlds). Consider the sentence "if $\neg A$ then ...": the closest $\neg A$-world must be a world of the context set, and it must satisfy $\neg A$, hence it has to be a B-world given the constraint on the assertion of disjunctions.

The debate between whether to deal with failures of IE and OI semantically or pragmatically is probably even more prominent with regard to SDA. Failure of the Stalnaker-Lewis semantics to satisfy SDA was pointed out early by several (Creary and Hill (1975), Nute (1975), Fine (1975), and Ellis et al. (1977)). At least three strategies have been explored to deal with SDA: one has been to question the Boolean status of "or" (see Loewer (1976), Van Rooij (2006), Alonso-Ovalle (2009)), but to maintain a non-monotonic semantics. The leading principle behind that approach is that "or" selects a set of alternatives, and that universal quantification over those is what effects the simplification (see Alonso-Ovalle for details). A distinct strategy, more along the lines of Stalnaker's treatment of OI, maintains a non-monotonic semantics and maintains that "or" is Boolean, but argues that conversational maxims require that the antecedent "if A or B" cannot select just the closest A-worlds, or just the closest B-worlds, but that it has to select both the closest A-worlds and the closest B-worlds, and any worlds in between (Klinedinst (2009)). Roughly, by saying "if Mary or Susan come to the party", the speaker has to mean something different from just "if Mary comes to the party" or "if Susan comes to the party". A third and more radical strategy is to forsake the Lewis-Stalnaker semantics entirely. Such a proposal has been made recently by Fine (2011), and involves adding more structure to the possible worlds framework than just similarity relation. Fine uses a version of situation semantics in which formulae are made true or false by states, with relations of exact and inexact verification and falsification. Essentially, his proposal is that $A > C$ is true at w iff whenever state t exactly verifies A, and u is a possible outcome of t relative to w, u inexactly verifies C. A disjunction is exactly verified by a state in his account iff the state exactly verifies one of the disjuncts or their conjunction. Finally, it should be noted that SDA is a valid inference rule under a plain strict conditional analysis of the conditional. Several authors (Warmbrōd (1981a), Lycan (2001)) see an additional argument in favor of the strict conditional analysis here.

All of these strategies, finally, also have to accommodate the fact that SDA does have some exceptions. A canonical example is that of McKay and Van Inwagen (1977):

(34) If Spain had fought with the Axis or the Allies, she would have fought with the Axis

In that case, the inference to “if Spain had fought with the Allies, she would have fought with the Axis” seems just wrong. Because of examples like this, all of the aforementioned strategies do rely on some pragmatic principles to explain away this inference. What the example suggests, here, is that one of the two alternatives mentioned in the antecedent is not on a par with the other. Fine (2011), for instance, considers SDA to be valid, but a sentence like (34) to be pragmatically odd because of this lack of symmetry between the disjuncts. Lycan sees the strict conditional analysis as capable of accounting for both valid and invalid cases of SDA (though, as pointed out above, the default is for it to validate SDA). Klinedinst’s approach, on the other hand, could explain that this is a case in which “if A or B” selects only one class of closest worlds.

In summary, IE, OI and SDA all are prima facie cases for the idea that the Stalnaker-Lewis analysis of conditionals fails to capture some valid inferences based on conditionals. In the case of OI and SDA, we see that the inference can be accommodated based on pragmatic principles concerning the use of disjunction. In the case of IE, the problem appears to be syntactic, more than pragmatic, namely to concern the fact that right-nested if-clauses are added up to the set of hypotheses relevant to evaluate the antecedent.

1.5 Referential vs. quantificational analyses

In this section we discuss two refinements of the framework proposed by Stalnaker and by Lewis. Both approaches provide a compromise between Stalnaker's semantics and Lewis's semantics by discarding the Uniqueness Assumption, but in retaining the Limit assumption. On the first approach, which we can call the *referential treatment* of if-clauses, the truth conditions for conditionals remain essentially those given in Stalnaker (1968), except that selection functions now are plural choice functions, that is, relative to an antecedent and world they select a set of worlds instead of a unique world. Empirical arguments in favor of this approach are put forward by Schlenker (2004) in particular, based on a comparison between the semantic behavior of "if"-clauses and the semantic behavior of plural definite descriptions. The second variant we have in mind, which we can call the *quantificational treatment* of if-clauses, consists in treating conditionals literally as strict conditionals, but coming with a mechanism of variable domain restriction. This approach has been advocated in particular by Warmbrōd (1981b), von Fintel (2001) and Lycan (2001).

Before examining the main differences between the two approaches, we should highlight what they have in common. The common part is the idea that 'if A, C' is true iff C holds in all the closest A-worlds to the actual world. That is, the two approaches will end up predicting identical truth conditions. The two approaches essentially differ concerning the syntax-semantics interface. On the plural choice function analysis defended by Schlenker (2004), the logical form of a conditional 'if A, C' is implicitly as follows:

(35) $[\iota W : A(W)](\forall w : W(w))(C(w))$: the (closest) A-worlds, they are such that every one of them is a C-world

On the strict conditional analysis with changing domain restriction, the logical form rather is as follows:

(36) $(\forall w : R(w))[A(w) \supset C(w)]$: for every world of the restricted domain, either A is not true there or C is

In (36), R is a parameter that acts as a domain restrictor. Lycan, for that matter, calls this kind of analysis a 'parametrically strict conditional'; it may equally be called a contextualized strict conditional. Importantly, this approach need not presuppose that relative to any given world, there is a unique A-world. Because of that, the approach also invalidate CEM, like the standard strict conditional analysis.

Both approaches can account for the non-monotonic behavior of conditionals. However, this non-monotonic behavior results from different mechanisms. On Schlenker's approach, non-monotonicity is a property typical of the behavior of definite descriptions, as first observed by Lewis (1973) himself. Indeed, definite descriptions can be seen to fail analogues of contraposition, transitivity and strengthening of the antecedent, since in particular:

(37) a. $[The\ A][C] \nvDash [The\ not\ C][not\ A]$
b. $[The\ A][B], [The\ B][C] \nvDash [The\ A][C]$
c. $[The\ A][C] \nvDash [The\ AB][C]$

For example, Lewis's original example for the counterpart of failure of strengthening is:

(38) The pig is grunting, but the pig with floppy ears is not grunting.

The first occurrence of 'the pig' is taken to denote the most salient pig, while the second occurrence, modified with 'with floppy ears', denotes a distinct pig. On the referential analysis of definite descriptions proposed by Schlenker, non-monotonicity directly results from the fact that definite descriptions rely on a salience ordering that varies with the property at issue.[11]

On the parametric strict conditional analysis, the non-monotonic behavior of conditionals is essentially the effect of the changing domain restriction of the universally quantified variable. One of the advantages of handling conditionals as parametrized strict conditionals in this way is also that it gives more flexibility than either Lewis's or Stalnaker's semantics to deal with non-monotonicity. Consider the following example:

(39) If John is with Mary, then he is not with Susan. So if John is with Susan, he is not with Mary.

This arguably is a case in which contraposition holds. The way it can be predicted is by letting the domain restriction remain constant from one conditional to the next. Contrast this with example (14). The latter arguably corresponds to a case in which each antecedent sets a different domain restriction:

(40) a. If Goethe had lived past 1832, he would be dead now. But if Goethe were not dead today, he would have lived past 1832.
b. $(\forall w : R(w))[A(w) \supset C(w)]$. $(\forall w : R'(w)[\neg C(w) \supset A(w)]$

[11] It should be stressed however that Schlenker does not argue so much for nonmonotonicity in his paper as much as for the idea that *if* is the counterpart of *the* in the modal domain. He writes: "even if the monotonic analysis of conditionals is correct, if-clauses should be analyzed as (monotonic) plural descriptions rather than as structures of universal quantification".

In the first conditional, $R(w)$ can be taken to select worlds in which every individual lives less than 120 years after his or her birth. In the second conditional, $R'(w)$ now selects worlds in which no limitation is set on the life span of individuals. This strategy, which underlies Warmbrōd and Lycan's approach to conditionals, is at the bottom of the dynamic approach of counterfactuals presented by von Fintel (2001). Part of the motivation for von Fintel's approach precisely concerns monotonicity issues, since von Fintel observes that NPI, which are typically assumed to be licenced in downward entailing environments (Ladusaw-Fauconnier generalization), can appear in the antecedents of conditionals, as in:

(41) If you had left any later, you would have missed the plane.

As pointed out by von Fintel, if conditionals are assumed to be non-monotonic, we lose a connection between NPI-licensing and downward entailingness. One way of recovering it is to maintain that conditional are monotone within limits specified by contextual domain restrictions (see von Fintel (1999)). It is less clear whether the referential analysis can accommodate the same limited form of downward entailingness for conditionals.

An additional argument in favor of the analysis of if-clauses either as contextually restricted universal quantifiers, or as definite descriptions, is that either of them can accommodate the combination of "if" with particles such as "only" and "even". "Even" and "only" are words that can combine with definite descriptions, as in "even the pig", or "only the pig". On the account of "if"-clauses as plural definite descriptions, the analysis of "only if" and "even if" can follow the same path as the analysis of "even" and "only" prefixing determiner phrases. In the case of the parametric strict conditional analysis, Lycan (2001: 17) puts forward as an advantage of the theory the fact that assuming "if *A*, *C*" means "*C* in any event in which *A*", then "even if *A*, *C*" means "*C* in any event including any in which *A*" and "only if *A*, *C*" means "*C* in no event other than one in which *A*'.

One of the issues regarding the choice between the analysis of 'if'-clauses as referential definite descriptions vs. contextually restricted universal quantifiers however concerns compositionality. Lycan, for example, does not provide a compositional derivation of constructions such as "even if" and "only if", but such an analysis would appear quite natural under the view of if-clauses as descriptions.[12] Secondly, the mechanism by which the restriction is set in a conditional appears to differ depending on the case: in (40-a), for instance,

[12] See Guerzoni and Lim (2007) for a compositional analysis of "even if" and a discussion of Lycan's view.

$R(w)$ is a default interpretation; in (40-b), by contrast, $R'(w)$ appears to be forced by the antecedent itself. Thirdly, failures of compositionality may generalize. Schlenker points out that the analysis of iterated conditional sentences, such as (42) below, "involves a drastic rearrangement of various parts of the sentence" if 'if' is analyzed as a universal quantifier, though follows a natural order under the referential analysis:

(42) If John comes, if Mary comes as well, the party will probably be a disaster.

Additional arguments finally are given by Schlenker for the referential account, in relation to the analysis of "then" in if-clauses (see below section 1.9).

In summary, both the contextualized strict conditional analysis and the referential analysis of if-clauses as plural definite descriptions offer different ways of amending and restructuring the Stalnaker-Lewis semantics for conditionals. The main difference between them concerns mostly compositionality, and the articulation between monotonicity and contextual domain restrictions.

1.6 If-clauses as restrictors

Whether as a material conditional, a strict conditional, or a variably strict conditional, logical theories of the conditional treat the conditional construction "if...then" by means of a binary sentential connective. In that regard, they make no syntactic difference between "or", "and", and "if". This does not mean that such theories are necessarily inadequate semantically, but they do not care about the syntax-semantics interface.[13] Several arguments can be given to show that coordination by means of "and" or "or" and subordination by means of "if" do not have the same syntax, however (see Geis (1970), Geis (1985), Lycan (2001), Bhatt and Pancheva (2006)). For instance, if-clauses can appear both sentence-initially and sentence-finally, but not so for "and" or "or":

(43) a. Joe left if Mary left.
b. If Mary left Joe left.

(44) a. Joe left and/or Mary left.
b. *and/or Mary left Joe left.

We saw that "if" can follow "even" or "only", but not so for "and" and "or":

[13] This does not include the referential and quantificational theories discussed in the last section, which, as linguistic theories, do care about this articulation.

(45) a. Lee will give you five dollars even if you bother him.
b. *Lee will give you five dollars even and/or you bother him.

More generally, there is a substantial body of evidence that if-clauses behave as adverbials, and that "then" is a pronominal adverb (see Iatridou (1993), Lycan (2001), Bhatt and Pancheva (2006); we return to "then" in section 1.9 below). The details of how such a view should be articulated vary between theories. Lycan, for instance, presents his version of the strict conditional analysis as an adequate way of dealing with subordination. In this section, we focus on the so-called Lewis-Kratzer analysis, which is based on separate evidence for the view that "if" behaves different from "or" or "and". Lewis (1975) considered a range of constructions with if-clauses, in which if-clauses restrict adverbs of quantification, as in:

(46) a. Always, if it rains, it gets cold.
b. Sometimes, it it rains, it gets cold.
c. Most of the time, if it rains, it gets cold.

Whereas (46)-a and (46)-b can be paraphrased in first-order logic as $\forall t(R(t) \supset C(t))$ and $\exists t(R(t) \wedge C(t))$ respectively, (46)-c cannot be paraphrased by means of a unary operator $Most$ taking scope either over or under a material conditional or a conjunction (see Kratzer (1991); von Fintel (1998b)). For instance, suppose $Most\ t\ A(t)$ is true exactly if the cardinality of the set of individuals that are A is greater than the cardinality of the set of individuals that are not A. And assume that restricted or binary "most", which we will write down as $[Most\ t : A(t)][C(t)]$, is true exactly if there are more AC individuals than $A\neg C$ individuals. It is easy to see then that $Most\ t(R(t) \wedge C(t))$, which says that most times are times at which it rains and it is cold, is stronger than what is intended by $[Most\ t : A(t)][C(t)]$. Conversely, $Most\ t(R(t) \supset C(t))$ is weaker, since it says that most times are either times where it does not rain, or times when it is cold. Quite generally, it was proved by Kaplan and by Barwise and Cooper (1981) that the binary operator "most" cannot be expressed by unary "most" taking scope over a Boolean formula, unlike what happens for the restriction of "every", which is expressible by the material conditional, or the restriction of "some" which is expressible by means of conjunction. As a matter of fact, even when we consider only the restriction of quantifiers such as "some" and "every", no uniform Boolean operator can be used to express their restriction.

Taken together, these elements indicate that the operation of quantifier restriction is not adequately handled by means of a binary sentential connective in classical logic. Based on these considerations, Lewis pointed out that for adverbs of quantification more generally, "the if of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts" (Lewis (1975): 14). Kratzer (1986), Kratzer (1991)) has offered to generalize Lewis's arguments. Her account consists in three related theses, which are i) "There is no two-place if...then connective in the logical forms of natural languages", ii) "If-clauses are devices for restricting the domains of various operators, and iii) "Whenever there is no explicit operator, we have to posit one." On Kratzer's account, a bare conditional such as "if John leaves, Mary will leave" is equivalent to "if John leaves, then it must be the case that Mary will leave", or "It must be the case that Mary leaves if John leaves". Whether the modal "must" is overt or covert, the function of the if-clause is to restrict its modal base (the set of worlds quantified over by "must").

One of the strengths of the Lewis-Kratzer account is that it derives the equivalence felt between sentences in which the operator appears above or below "if":

(47) a. There is one chance in three that if you throw an even number, it will be a six.
b. If you throw an even number, there is one in three that it will be a six.

(47)-b, in that case, can be assumed to result from the logical form [there is one chance in three: even][six] by movement (with 'even' as restrictor, and 'six' as the nuclear scope of the operator [there is one chance in three]).[14] Similarly, it can be used to provide a direct justification of the law of import-export, namely the fact that right-nested if-clauses can be expressed by conjoining their antecedents into one antecedent. Specifically, the Lewis-Kratzer analysis of if-clauses is also particularly appropriate regarding the interaction of if-clauses with probability operators (see Kratzer (1991), Egré and Cozic (2011)). Grice (1989), for instance, considers a scenario in which Yog played 100 games of chess, played White on 90 of those 100 games, won exactly 80 of those when he had white, and lost all of ten games in which he had black. Grice considers the case in which a speaker talks about the last game, not knowing whether Yog had white or black. This is a case in which the speaker can truthfully say:

(48) a. There is a probability of 8/9 that if Yog had white, he won
b. There is a probability of 1/2 that if Yog lost, he had black
c. There is a probability of 9/10 that either Yog didn't have white or he won

[14] Huitink (2007) presents two different compositional implementations of the Lewis-Kratzer analysis, one by von Stechow, the other by von Fintel.

Clearly, we would not be able to explain the joint consistency of all three claims if each of the operators "there is a probability of *x*" was taking scope over a sentence analyzed as a material conditional, for all three are equivalent under the material conditional analysis. On the other hand, we get the right truth conditions if the if-clause, in the first two examples, is assumed to restrict a probability operator (or identically, a proportional quantifier):

(49) a. $[8/9 : \textit{white}][\textit{win}]$
b. $[1/2 : \neg \textit{win}][\textit{black}]$
c. $[9/10][\neg \textit{white} \vee \textit{won}]$

As another example, assume a domain with exactly three worlds, *a*, *b* and *c*, with equal probability, and assume that *a* and *b* are all and only worlds in which it rains, and *a* is the only world in which it gets cold. This is a case in which one can say:

(50) There is a probability of 1/2 that if it rains, it gets cold.

This is a case in which, relative to the probability distribution under consideration, no Boolean proposition over the domain can receive a probability equal to half (there are 8 propositions expressible on the algebra, taking either probability 0, 1/3, 2/3, or 1). This indicates that the embedded sentence "if it rains, it gets cold" does not express any Boolean proposition in that case (see Hájek and Hall (1994), Adams (1998), Egré and Cozic (2011)).

Under the analysis of if-clauses as restrictors, however, adequate truth conditions for (50) are directly predicted. Importantly, this argument shows that the conditional probability that it rains, given that it is cold, cannot be equated to the probability of any two-valued proposition taken to represent the conditional sentence as a whole. This, in a nutshell, is the gist of Lewis's so-called triviality result for conditionals (Lewis (1976)), showing that one cannot in general equate the probability of a conditional sentence to the conditional probability of the consequent given the antecedent (an equation often called Adams' Thesis in the more recent literature).[15]

Several aspects of Kratzer's analysis have been recently criticized, however, in particular regarding the interaction of theses i) and iii). One argument discussed by von Fintel (2007), von Fintel (2011), and more recently by Huitink (2007) and Rothschild (forthcoming) concerns cases of anaphor, such as:

[15] The literature on Adams thesis and Lewis's triviality results is very large. See in particular Hájek and Hall (1994) for a technical survey, Bradley (2002) for a generalization of Lewis's result, Kaufmann (2009) for a recent comprehensive treatment of the probability of conditionals, and Douven and Verbrugge (2010) for an investigation of the empirical validity of Adams' thesis.

(51) a. If John leaves, Mary will leave.
b. I do not think so.

The argument given is that the anaphor introduced by "so" in (51)-b should refer back to the conditional sentence in (51)-a, with its silent necessity operator. However, this would appear to say that the person who denies (51)-a denies the whole modalized sentence expressed in (51)-a, which is too strong if the modality is an epistemic modality relative to the utterer of (51)-a. The way Kratzer's account can explain the example is either by assuming that the covert modal expresses necessity relative to both speakers, or that the anaphor picks up only the structural contribution of the antecedent and consequent to the proposition expressed in (51)-a (see von Fintel (2006)). Based on this and on further arguments, von Fintel, Huitink and Rothschild propose an alternative approach, namely to maintain the expression of "if...then" by means of an unmodalized binary sentential connective, but to give up on using a two-valued underlying semantics.[16] We discuss the prospects of such an account in the next section.

1.7 Trivalent analyses

An important reason to entertain a trivalent analysis for conditional sentences concerns the proviso that a material conditional sentence is classically true whenever its antecedent is false. A common consideration is that an indicative conditional whose antecedent is not true cannot be evaluated as true or false, and so remains indeterminate in truth value. Probably the first author to have made this proposal explicitly is De Finetti (1936), in a short paper about trivalent logic and conditional probability, where he contrasts two connectives, one called implication ($A \supset C$), and the other called subordination ($C|A$), with the following truth tables (columns are for the consequent C, and rows for the antecedent A):[17]

In the above tables, 1 stands for true, 0 for false, and 1/2 for the third truth value (which De Finetti writes N).[18] Although De Finetti gives few comments

[16] See also Yalcin (2012) on further elements of criticism of the restrictor view.

[17] We are indebted to Jean Baratgin and Guy Politzer for bringing De Finetti's paper to our notice. Milne (2012) points out that the idea of a defective truth table for the conditional was proposed independently by J. Schächter as early as 1935 (the same year of De Finetti's communication in Paris) and, in the area of psychology, by Wason in 1966.

[18] N stands for neither true nor false. We use 1/2 instead of N, following Łukasiewicz (1920), since it is handy to use it for the other connectives, to preserve that conjunction is the minimum, disjunction the maximum, and negation the distance from 1, over the set $\{1, 1/2, 0\}$.

$A \supset C$	1	1/2	0
1	1	1/2	0
1/2	1	1/2	1/2
0	1	1	1

$C\|A$	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	1/2
0	1/2	1/2	1/2

Table 1.1 *3-valued material conditional (left) vs. De Finetti's (right)*

on his tables, he motivates the one on the right by the consideration of bets. He gives the example of a bet on a running race supposed to take place the next morning. The target proposition is that if the race takes place, the winner will be so-and-so. De Finetti points out that if the race does not take place, the bet is called off.[19] The left table, for so-called implication, on the other hand, coincides with the table for the material conditional in Kleene's strong logic (or similarly, in Łukasiewicz logic). Unlike what De Finetti calls supposition, this conditional is exactly true when the antecedent is false, and when the consequent is true, thereby preserving the defining property of the bivalent material conditional.

De Finetti's table for so-called supposition has been put forward independently by Belnap (1970), and some of its recent advocates are McDermott (1996), Huitink (2008) and Rothschild (forthcoming).[20] Its main interest in relation to the considerations of the previous section is that it appears to be suitable to deal with restricted quantification by means of if-clauses.[21]

In order to deal with quantifier restriction, Belnap proposes the following truth conditions (the case of "most" is a generalization not directly in Belnap, but see Huitink (2008)):

(52) a. $[\![\forall x(Cx|Ax)]\!] \neq 1/2$ provided $[\![\exists x Ax]\!] = 1$. If so,

[19] De Finetti (1937) identifies conditional probabilities as the betting quotients for such conditional bets, and shows on the basis of a Dutch Book argument that these conditional probabilities should obey the so-called ratio formula $P(A|B) = P(A \wedge B)/P(B)$. On that occasion, de Finetti elaborates an algebra, later called "conditional event algebra", which is analogous to the trivalent logic described above. Milne (1997) provides an extensive philosophical and historical study of De Finetti's approach to conditionals.

[20] Belnap's original semantics for the conditional was soon thereafter modified in Belnap (1973), where Belnap no longer assumes that a conditional has to be defective whenever its antecedent is false. To prevent confusion, we will refer to this conditional primarily as the De Finetti-Belnap conditional, or simply as De Finetti's conditional (see Table 1.10).

[21] This feature, explored by Belnap, was mentioned in a footnote by Lewis (1975) as an alternative to the treatment of if-clauses as restrictors. von Fintel (2006) should be credited for bringing attention to Lewis's remark in the last few years, and for opening again the debate about the validity of the Lewis-Kratzer analysis. Interest for the defective truth table also looms quite large in the literature on the psychology of conditionals. See in particular Evans et al. (1993), Politzer et al. (2010), and further references in Milne (2012).

$[\![\forall x(Cx|Ax)]\!]$ is the minimum of the set of $[\![\text{Cd}]\!]$ for every d such that $[\![\text{Ad}]\!] = 1$.

b. $[\![\exists x(Cx|Ax)]\!] \neq 1/2$ provided $[\![\exists xAx]\!] = 1$. If so, $[\![\exists x(Cx|Ax)]\!]$ is the maximum of the set of $[\![Cd]\!]$ for every d such that $[\![Ad]\!] = 1$.

c. $[\![Most\, x(Cx|Ax)]\!] \neq 1/2$ provided $[\![\exists xAx]\!] = 1$. If so, $[\![Most\, x(Cx|Ax)]\!] = 1$ if among individuals d for which $[\![Ad]\!] = 1$, most of them are such that $[\![Cd]\!] = 1$; it is 0 otherwise.

By this mechanism, restriction can be handled uniformly for all three quantifiers considered in the previous section. In particular, (52)-a is true provided all of the As are Cs. It is false if some A is not C. The sentence is indeterminate if all no individual is A. (52)-b, similarly is true exactly when the sentence would be true in the bivalent case, is indeterminate if there is no A or no C, and is false if there is an A and every such A is not C. (52)-c, it should be noted, now provides a way of restricting unary *most* by means of the suppositional connective.

Another application of the De Finetti-Belnap table concerns the treatment of conditional sentences in the scope of probability operators (see McDermott (1996), Rothschild (forthcoming)). We saw in the previous section that in some cases, given a probability distribution, a sentence like "there is a probability of 1/2 that if it rains, it gets cold", cannot be expressed by the assignment of a probability to any possible-world proposition. We explained in what sense this can be considered as evidence for the Lewis-Kratzer analysis of if-clauses. However, several philosophers, based on Lewis's triviality result, have defended the view that conditionals should not be seen as having truth conditions for that matter (in particular Adams (1965) Gibbard (1981), and Edgington (1995), all representatives of the so-called 'No Truth Value' view of conditionals). The trivalent analysis offers an alternative to both views. De Finetti (1936) introduced three-valued logic precisely to account for the logic of conditional probability, and McDermott (1996) and Rothschild (forthcoming) consider the following notion of probability assignment for a conditional expressed by $(C|A)$. Let $[\![\phi]\!]_1$ be the set of worlds in which ϕ takes value 1, and $[\![\phi]\!]_0$ be the set of worlds in which ϕ takes the value 0. Given a probability distribution p over the sentences of a propositional language with the De Finetti/Belnap suppositional connective, define an extended probability assignment to be the function p' such that $p'(\phi) = \frac{p([\![A]\!]_1)}{p([\![A]\!]_1 \cup [\![A]\!]_0)}$. This is the probability that A is true, given that it is defined. As shown by McDermott and Rothschild, this can be used to capture conditional probabilities in terms of the probability of the suppositional conditional. For instance, consider again the domain W with three worlds a, b, c with equal probability, and such that $[\![\text{A}]\!]_1 = \{a, c\}$ and $[\![\text{C}]\!]_1 = \{a\}$. One can check that $p'(C|A) = p([\![C \wedge A]\!]_1/p([\![\text{A}]\!]_1) = 1/2$, that is, the probability of the conditional is equal to the conditional probability.

Some further benefits of this trivalent analysis are worth pointing out. In particular, the law of import-export is a valid law for the De Finetti conditional, assuming the Strong Kleene analysis of conjunction, and defining validity as preservation of the value 1 from premises to conclusion. Likewise, the paradoxes of material implication are not valid schemata, and neither is strengthening of the antecedent, nor contraposition. Transitivity holds, however, indicating that the latter three schemata need not hold or fall together. Likewise, $\neg(C|A)$ and $(\neg C|A)$ are equivalent under that definition of validity.

Not all of the desirable features of the indicative conditional are retained, however, under that definition of validity. First of all, defining validity in terms of preservation of the value 1 overgenerates: it implies that from "if A then C", one can validly infer "if C then A", a fallacy known as the Affirmation of the Consequent. There are several ways around this problem. One way is to maintain the standard definition of validity, but to constrain the trivalent truth-table of the conditional so as to avoid the problem (viz. Farrell (1979), whose truth-table coincides with De Finetti's, except that $v(C|A) = 0$ when $v(A) = 1/2$ and $v(C) = 0$). Another approach is to modify the usual definition of logical consequence. Three-valued logic offers more options than two-valued logic for that: besides validity as preservation of the value 1, one can define validity as preservation of the value > 0. As applied to the strong Kleene valuation scheme for disjunction, conjunction and negation, the latter definition gives the logic LP (Priest's 'Logic of Paradox', Priest (1979)), which is dual to Kleene's strong logic (standardly called K3, see Priest (2008) for an overview of both logics).

The definition of validity as preservation of values > 0 from premises to conclusion blocks the Fallacy of Affirming the Consequent for De Finetti's scheme. It preserves the equivalence between wide scope negation and narrow scope negation and the law of import-export, and it invalidates transitivity. But this time it also validates some problematic schemata, in particular the paradoxes of material implication, as well as contraposition and antecedent strengthening. Because of that, one influential choice in relation to De Finetti's schema (for instance (McDermott (1996), section 4) has been to define validity by requiring both the preservation of value 1 from premises to conclusion, *and* as the preservation of value 0 from conclusion to premises. As applied to the Strong Kleene connectives, this corresponds to taking the intersection of K3 and LP, a logic sometimes called S3 (for instance in Field (2008), for 'symmetric Kleene logic'), since preserving value 0 from conclusions to premises is equivalent to preserving value > 0 from premises to conclusion. Under this combined definition of validity, only the schema of Conditional Negation and Import-Export remain valid among the main schemata we have discussed (see the Appendix, Table 1.10).

For McDermott, an additional motivation for the choice of this definition of logical consequence is that it makes a bridge between a probabilistic notion of validity (along the lines of Adams definition of p-validity, see Adams (1998)), intended to mirror the degree to which a proposition is accepted or assertable, and the trivalent notion of validity, in the form of the following sufficiency condition: if A logically implies C, then $p'(A) \leq p'(C)$, where $p'(\phi)$ is defined as above as the extended probability of ϕ.[22]

The choice of the S3-schema does not avoid an undergeneration problem, however, depending on how some inferences are viewed. In particular, modus ponens and modus tollens are not valid for De Finetti's conditional, and other standard schemata are lost for the other connectives (in particular Disjunctive Syllogism, the inference from $A \vee C$ and $\neg A$ to C). The Or-to-If inference from $A \vee C$ to $(C|\neg A)$ is not valid either in case A is true, and the reader can check that Simplification of Disjunctive Antecedents is not valid in case exactly one of the disjuncts is undefined.[23] A proposal made by Huitink (assuming validity to be value 1-preservation) is that Or-to-If can be pragmatically regained in terms of Strawson-entailment (see Strawson (1952), Belnap (1973), von Fintel (1999)), namely for the case where one assumes that $(C|\neg A)$ is either true or false, which forces $\neg A$ to be true. This suggests that, even in the domain of trivalent analyses, some pragmatic machinery may need to be brought in to deal with specific inferences. On the other hand, appeal to Strawson-entailment may be too powerful a mechanism in the case of conditionals (as opposed to presupposition, cf. Belnap (1973)), since it would actually rescue all the patterns listed as invalid under the original scheme for consequence (see Table 1.10).[24]

To conclude on this section, we should point out that several other partial and

22 Let $[\![A]\!]_1 \subseteq [\![C]\!]_1$, and $[\![C]\!]_0 \subseteq [\![A]\!]_0$, and let $p([\![A]\!]_i) = a_i$, for $i = 1, 1/2, 0$, and likewise $p([\![C]\!]_i) = c_i$. Then, $a_1 \leq c_1$, and $c_0 \leq a_0$, which implies that $\frac{a_1}{a_1+a_0} \leq \frac{c_1}{c_1+c_0}$, that is $p'(A) \leq p'(C)$.

23 Both problems are averted if validity is defined only as preservation of the value > 0, but not the modus ponens and modus tollens cases

24 According to Huitink (p.c.): "Strawson-entailment doesn't apply across the board, but only when the premises and conclusion can *consistently* be assumed to have a truth-value given that the premises are true." An example Huitink gives to illustrate this point is: "Or-to-if is also Strawson-valid, but Strawson-entailment places a cut between ordinary cases (where "A or C" is true and both A and C are still open) and those where 'A or C' is grounded in A. In the latter case, "if not-A then C" cannot be defined."

trivalent analyses have been suggested for the conditional (see Bradley (2002), Cantwell (2008) a.o.), as well as many-valued semantics more generally (see Stalnaker and Jeffrey (1994), Kaufmann (2009)). Giving an overview of those would be beyond the scope of this chapter: even for the case of three-valued conditionals, we can see that the definition of what counts as a good conditional depends not only on the choice of a particular valuation scheme for the connectives, but again on what counts as an adequate definition of validity for a given scheme. Another pressing issue for trivalent approaches, moreover, concerns the treatment of counterfactual conditionals. For three-valued approaches as well as for two-valued approaches, the issue of non-truth-functionality raised by Quine (1950) appears to hold equally (all the more since, obviously, not all counterfactual conditionals should be considered undefined in truth-value on the grounds that their antecedent is contrary to fact). This, however, is not necessarily an argument against a three-valued approach, but the indication that more machinery needs to be adduced to deal with tense and mood proper.[25]

1.8 Indicative vs. counterfactual conditionals

A central division within conditional sentences is the division between so-called *indicative* and *subjunctive* conditionals. A classic example of the division is Adams' pair (Adams (1970):

(53) a. If Oswald did not kill Kennedy, then someone else did.
 b. If Oswald had not killed Kennedy, then someone else would have.

As pointed out by Adams, the two sentences do not have the same truth conditions. (53)-a is true given what we know about Kennedy's death, namely that Kennedy was killed by someone. (53)-b is not obviously true, however, since even under the additional assumption that Kennedy was actually killed by Oswald, the sentence will only be true to someone who believes there was a conspiracy to kill Kennedy. (53)-a is called an indicative conditional sentence: in particular, it uses past indicative both in the antecedent and consequent. (53)-b is called a subjunctive conditional: in that case it uses the past in the antecedent, and subjunctive 'would' in the consequent. A clearer illustration of the difference is given by cases in which the English subjunctive appears both in the antecedent and consequent, as in:

[25] See Farrell (1979) for a discussion of three-valued logic in connection to counterfactual conditionals. Farrell basically argues that modal operators need to be added in to handle counterfactuals in that framework.

(54) a. If Mary were rich, she would be happy.

Iatridou (2000) defines the subjunctive mood as ‘the morphological paradigm that appears in the complement of verbs of volition and/or command’. In English, ‘were’ and ‘would’ both correspond to such paradigms, as evidenced by such sentences as: ‘I wish I were rich’, or ‘I wish it would rain’. So-called subjunctive conditionals are often associated to the expression of counterfactuality in language. However, it is widely agreed that this is a misnomer (Iatridou (2000), Kaufmann (2005)). Iatridou points out that some languages use the subjunctive to express counterfactuality (viz. German, Icelandic, Spanish, Italian), whereas other languages simply do not have a separate subjunctive mood (Danish, Dutch), or some languages that have a subjunctive mood do not use it to express counterfactuals (viz. French). An illustration of the fact that in English, a conditional can be in the subjunctive without expressing a counterfactual conditional is given by Anderson’s example (Anderson (1951)):

(55) If the patient had taken arsenic, he would show exactly the same symptoms that he does in fact show.

In uttering (55), the speaker indicates that he considers as an open possibility that the patient took arsenic. From a semantic point of view therefore, it is more adequate to talk of a division between counterfactual and non-counterfactual conditionals, to distinguish between two kinds of cases: those in which the antecedent is assumed to be false in the context, and those in which it is not assumed to be false. The distinction subjunctive-indicative is morphological rather than semantic. In English, however, the relation between morphology and counterfactuality seems to be the following: *the subjunctive is necessary to express counterfactuality*, or equivalently, *the expression of counterfactuality bans the indicative*, that is, indicative implies non-counterfactuality. However, *the use of the subjunctive is not sufficient to express counterfactuality*.

Ever since the work of Goodman (1947), the question has been asked whether a unified semantics can be given for indicative and counterfactual conditionals. Lewis (1973) famously considered indicative conditionals and counterfactual conditionals to have distinct truth conditions (for him, in particular, indicative conditionals had the truth conditions of the material conditional). An opposing view is expressed in the work of Stalnaker, who basically considers that indicative and counterfactual conditionals have identical truth conditions, but that they come with different pragmatic presuppositions. In this section we first briefly explain the main ingredients of Stalnaker’s account, and then give an overview of its relation with more recent work about the interaction between tense and mood in conditionals.

The way Stalnaker (1975) accounts for the division between indicative and counterfactual conditionals is by the same mechanism used to account for simplification of disjunctive antecedents, namely by means of the constraint on selection functions exposed in (33), that "if the conditional is evaluated at a world in the context set, then the world selected must, if possible, be within the context set as well". Let S denote the context set, namely the set of propositions mutually believed by the participants to the conversation, and let $f(S, \phi)$ denote the set of worlds $f(w, \phi)$ such that $w \in S$, that is the set of closest ϕ-worlds to the worlds in the context set. More formally, Stalnaker's selection constraint is expressible as follows (see von Fintel (1998a)):

(56) if $S \cap A \neq \emptyset$, then $f(S, A) \subseteq S$

Consider A to be some antecedent if-clause, then the constraint says that if the antecedent is compatible with the context set, the closest world to the antecedent will also be in the context set. An illustration of Stalnaker's constraint can be given on the indicative Oswald case (53-b). Here the shared presupposition between speakers is that Kennedy was killed (K), that is: $S \subseteq K$. Clearly, the proposition that Kennedy was killed can be decomposed into worlds where Oswald was the killer (O) and worlds where someone else was the killer (E). By the selection constraint, $f(\neg O, S) \subseteq S$, hence $f(\neg O, S) \subseteq K$. Moreover, by clause 1 of Stalnaker's semantics, $f(\neg O, S) \subseteq \neg O$, hence $f(\neg O, S) \subseteq E$, which means that the closest world where Oswald did not kill Kennedy is a world where someone else did.

In contrast to the case of indicative conditionals, Stalnaker makes the hypothesis that "the subjunctive mood in English and some other languages is a conventional device for indicating that presuppositions are being suspended, which means in the case of subjunctive conditional statements that the selection function is one that may reach outside the context set". This corresponds to the following feature of the context:

(57) When subjunctive is used, possibly $f(A, S) \nsubseteq S$

To illustrate it, assume that it is now presupposed that Oswald actually killed Kennedy, that is $S \subseteq O$. Necessarily $f(\neg O, S) \subseteq \neg O$, hence $f(\neg O, S) \subseteq \neg S$. In that case, the closest world where Oswald did not kill Kennedy must be outside of the context set, but this can be a world where someone else kills Kennedy, like a world where Kennedy is not killed.

An important aspect of Stalnaker's statement of the effect of using the subjunctive mood is that the use of the subjunctive does not always force to go outside of the context set. An example given by Stalnaker concerns the following piece of reasoning:

(58) The murderer used an ice pick. But if the butler had done it, he wouldn't have used an ice pick. So the butler did not do it.

At the moment the conditional is uttered, it cannot be presupposed that the butler did not do it, for otherwise the conclusion of the argument would be redundant, hence the conditional is not counterfactual. Thus this is a case in which the context set is compatible with both options regarding the butler. As pointed out by Stalnaker and as emphasized by von Fintel, the same reasoning in the indicative sounds odd, however:

(59) The murderer used an ice pick. (?) But if the butler did it, he didn't use an ice pick. So the butler didn't do it.

The problem here is that by the selection constraint, the closest world where the butler did it should be a world where he used an ice pick, an assumption contradicted by the consequent. Hence, the use of the subjunctive is more appropriate here, but we see that it does not dictate to always go outside the context set.

Stalnaker's hypothesis gives a pragmatic answer to the question of how the choice is made between indicative and subjunctive conditionals. Another difficult question concerns the interaction of tense and mood in the expression of indicative and subjunctive conditionals. An observation common to all accounts of counterfactuals, in particular, is that they use past morphology. Consider the following pair (adapted from Schulz (2007)):

(60) a. If Peter took the plane, he would be in Frankfurt this evening.
b. If Peter took the plane, he must be in Frankfurt by now.

In the indicative conditional (60)-b, the antecedent refers to a past event relative to the utterance time of the sentence. This is not so in (60)-a, where the speaker means that if Peter were to take the plane *now* or even possibly at some moment between now and the evening, he would be in Frankfurt in the evening. In that case, the past tense does not appear to play its temporal function, namely to refer to a past moment relative to the utterance situation. This feature has been given considerable attention in a number of recent works (see Iatridou (2000), Ippolito (2003), Asher and McCready (2007), Schulz (2007)). We cannot hope to discuss all of those accounts here (see Schulz (2007), chapter 6 for an extended survey). To make the connection with Stalnaker's account of the indicative/subjunctive division, however, we find it worthwhile to give a brief overview of Iatridou's account. On Iatridou's account the use of the past in counterfactuals is essentially modal, and not temporal. The mechanism behind the temporal and the modal understanding of the past is essentially the same however. In the temporal case, Iatridou points out that the past signals precedence, namely that the topic time $T(t)$ (temporal interval for the event talked about in t) of the event reported is located before the utterance time $S(t)$ (relevant interval of the utterance). In the modal case, by analogy, Iatridou distinguishes between the set of topic worlds $T(w)$, the worlds talked about, and the context set $S(w)$, namely the worlds of the speaker. In that case, the past signals exclusion of topic worlds from the context set. Whether as temporal or modal, however, Iatridou does not see the exclusion triggered by the past as semantically mandated, but rather she views it as cancellable and optional in some contexts (in agreement with Anderson's example in the modal case).

A further generalization proposed by Iatridou is that "when the temporal coordinates of an event are not set with respect to the utterance time, morphology is always Imperfect." For instance, the pair (60)-a vs. (60)-b would translate in French as:

(61) a. Si Pierre prenait l'avion, il serait à Francfort ce soir.
b. Si Pierre a pris l'avion, il doit être à Francfort ce soir.

In (61)-a, "prenait" is the imperfect past (*imparfait de l'indicatif*), whereas "a pris" in (61)-b is the perfect past (*passé composé*). The same analysis of the past as exclusion works for modals like 'would' and 'might' in English, seen as past forms of the modal roots 'woll' and 'moll', and Iatridou shows that it can be extended to other paradigms. In the case of the French conditional mood, for instance, the latter can be seen as being formed of past imperfect morphology appended to a future morpheme, as evidence by the contrast:

(62) Si tu viens, tu aime-r-as la ville.
If you come-IND-PRES, you like-FUT-2nd-sg-PRES the city
'If you come, you will like the city.'

(63) Si tu venais, tu aime-r-ais la ville.
If you come-IMP-PAST, you like-FUT-2nd-sg-IMP-PAST the city
'If you came, you would like the city'

The interaction of tense and mood becomes more involved with the so-called past irrealis, namely for sentences such as:

(64) Si tu étais venu, tu aurais aimé la ville
If you were come, you have-would liked the city
'If you had come, you would have liked the city'

Typically, a sentence in this form refers to a counterfactual possibility relative to a past topic time. So the past tense in the antecedent can refer to a past event. Moreover, the use of the pluperfect is needed to express that this counterfactual possibility is relative to the past. However, the sentence can occasionally be used in the same sense as: 'if you could have come (tomorrow), you would have liked the city', but with the implicature that the addressee has settled not to come.

1.9 Relevance conditionals

To conclude this chapter, we turn to the problem of the semantic analysis of relevance conditionals such as (7), a variant of which is repeated here as (65):

(65) There are biscuits on the sideboard if you're hungry

These conditionals, also called *biscuit conditionals*, *Austin conditionals*, and *nonconditional conditionals* (see Geis and Lycan (1993), Siegel (2006)) pose a problem too for the prospect of having a unified analysis of conditional sentences, since, unlike standard indicative conditionals, uttering a relevance conditional appears to be essentially asserting the consequent, without letting the truth of the consequent depend on the truth of the antecedent. Like the literature on counterfactual conditionals, the literature on biscuit conditionals is too large for us to do justice to the topic. However, we wish to highlight at least two aspects of the research on these conditionals.

The first aspect, mentioned above in the introduction, is that relevance conditionals differ from standard indicative conditionals by the exclusion of the adverb 'then' in the consequent. An important contribution on the distribution of 'then' in conditional sentences more generally is Iatridou (1993). Iatridou uses as background theory the Lewis-Kratzer analysis of if-clauses, that is she supposes that bare conditionals come with a silent operator *Must*. Her proposal is that use of 'then' in the consequent of a conditional sentence 'if A, C' introduces an assertion and a presupposition, as follows:

(66) a. Assertion: $[Must : A][C]$
b. Presupposition: $\neg[Must : \neg A][C]$

For example, under a strict conditional analysis, an indicative conditional such as: 'if John visits, then Mary will be happy' would assert that in all the worlds in which John visits, Mary is happy, and would presuppose that in some of the worlds in which John does not visit, Mary is not happy. Iatridou's hypothesis is highly predictive, in particular it can explain oddities of the use of 'then' for many cases where the presupposition is violated, as in:

(67) If Bill is dead or alive, (*then) John will find him.

In a relevance conditional like (65), similarly, Iatridou points out that the insertion of 'then' should imply that in some of the worlds in which you are not hungry, there are no biscuits on the sideboard. However, such a meaning is precisely ruled out in this case, since the presence or absence of biscuits on the sideboard is not meant to depend on the addressee's hunger condition.

Further arguments have been adduced in favor of Iatridou's analysis of 'then', in fact in support of the referential analysis of if-clauses, rather than on the restrictor analysis originally used by Iatridou. Bittner (2001), Schlenker (2004) and Bhatt and Pancheva (2006) treat conditionals as correlative constructions, with if-clauses analyzed as free relatives in the position of sentence topics, and with 'then' as a world pronoun acting as a pro form. A central element of analogy between if-clauses in the modal domain and topicalization in the individual domain concerns left-dislocated constructions in German. Iatridou gives the following example from German:

(68) Hans, der hat es verstanden
Hans, he has it understood.

In this case, the sentence asserts that Hans has understood, and either presupposes or implicates that other people have failed to understand. Thus, the pronoun 'der' in this construction appears to behave exactly like 'then' in the modal domain. The connection between conditionals and left dislocated constructions in German has recently been used by Ebert et al. (2008) to propose a unified analysis of indicative conditionals and biscuit conditionals. Ebert et al. point out that German distinguishes between two kinds of topic constructions, German left dislocation (LD) and hanging topic left dislocation (HTLD):

(69) Den Pfarrer, den kann keiner leiden. (GLD)
The pastor, him can noone bear.

(70) Der/den Pfarrer, keiner kann ihn leiden. (HTLD)
The pastor, noone can him bear

In agreement with the referential analysis of Schlenker, they see some analogies between GLD and indicative conditionals. According to them, HTLD constructions presents corresponding analogies with biscuit conditionals. GLD and HTLD set up two different kinds of topicality on their analysis, *aboutness* topicality for GLD, and *frame setting* topicality for HTLD. In the latter case, their analysis for (70) does not treat the pronoun "ihn" as directly bound to the topic, "den Pfarrer", contrary to what happens in (69), but rather, as a free variable, whose reference is linked to that of the topic only because the latter is made salient in the context. In contrast to that, they see the coreference as obligatory for GLD, in particular because the latter permits binding, unlike HTLD. This analogy enables Ebert et al. to give a parallel analysis of indicative conditionals vs. biscuit conditionals. They propose the following logical forms for an indicative conditionals and for its relevance counterpart:

(71) a. If you are looking for the captain, then he is here.
b. $REF_X(\iota_{w_0} w(looking(w)(listener)))$
$\wedge\ ASSERT(here(X)(captain))$

(72) a. If you are looking for the captain, he is here.
b. $REF_X(\iota_{w_0} w(looking(w)(listener)))$
$\wedge\ ASSERT(here(w_0)(captain))$

On their analysis, $REF_X(y)$ indicates the establishment of an act of topical reference of y as the topic X, $\iota_{w_0} wP(w)$ denotes 'the closest world w satisfying P relative to w_0 and $ASSERT(\phi)$ indicates the act of assertion of the sentence ϕ. The main difference between the two constructions is that in the biscuit case, the proposition asserted is anchored to the actual world, whereas in the indicative conditional case the proposition asserted picks up the referent introduced as the topic. In the second case, only the consequent of the conditional is asserted, and the antecedent only sets a topic relevant for the assertion proper. As shown by Ebert et al., this kind of frame setting happens in other sentences, such as:

(73) As for the pastor, the marriage sermon was wonderful.

Whether the referential analysis of if-clauses is really needed to capture the distinction proposed by Ebert et al. can be subjected to discussion. For instance, the way (71) is intelligible would typically be in some bound sense, as in: "whenever you are looking for the captain, he is here". The latter is readily analyzed as a case of universal quantification over time situations, and the temporal reference of the consequent is bound by the antecedent. Compare with: "Are you now looking for the captain? he is here". In that case, the temporal reference of the consequent has to be the present moment, and the question is only asked to make the assertion relevant.

Several other proposals have been made to deal with biscuit conditionals recently. Franke (2007) in particular outlines an explanation of why the antecedent of a biscuit conditional is believed based on purely pragmatic considerations about the epistemic status of the antecedent relative to the consequent. DeRose and Grandy (1999) suggest that Belnap's trivalent account of indicative conditionals can actually be accommodated to deal with biscuit conditionals. Siegel (2006) denies that the consequent of a biscuit conditional is always asserted as true by the speaker, and proposes a metalinguistic analysis of biscuit conditionals of the form 'if *A*, then there is a relevant assertion of 'C''. Although it is not the place to adjudicate the debate between these various proposals, it may be pointed out that Ebert et al.'s is probably the main one driven by the observation of the exclusion of 'then' in conditional sentences.

1.10 Perspectives

We have been deliberately partial and selective in building this chapter on conditionals, mostly with an aim to being self-contained, but omitting some important references and some aspects of the semantics of conditionals. We refer to von Fintel (2011) for another recent survey on conditionals with a partly different angle on the topic, providing further bibliographical indications, in particular concerning the link between conditionals and dynamic semantics, conditionals and epistemic modals, and further aspects of the interaction of tense and mood in conditionals.

We should mention two lines of research which lie at the frontiers of linguistic inquiry proper. The first bears on the relation between belief dynamics and conditionals, and is deeply influenced by the famous Ramsey Test (Ramsey (1929)) according to which the belief attitude towards "If A, then C" is determined by the belief attitude towards C after a rational belief change based on the supposition that A. [26] After several decades of intense scrutiny in different frameworks, the Ramsey Test is still an object of study (Bradley (2007), Dietz and Douven (2010), Hill (2012)). A second line of research on conditionals concerns the interface between the psychology of reasoning and the semantics and pragmatics of conditionals. Interest for the connection between conditional reasoning was sparked by Wason's famous selection task (Wason (1960)), the

[26] Adams' Thesis can be seen and has often be viewed as a probabilistic version of the Ramsey's Test, where conditionalization is assumed to be a rule of rational belief change.

origin of a very large literature on the verification biases for conditional sentences. Some more recent works on the psychology of hypothetical reasoning include Johnson-Laird and Byrne (2002), Over and Evans (2003), Over et al. (2007), Politzer and Bourmaud (2002), Politzer (2007)) and Douven and Verbrugge (2010).

Appendix

The following table summarizes the main frameworks and schemata examined in Sections 2 to 7, comparing which schemata are valid (+) or not (−) among Falsity of the Antecedent (FA), Truth of the Consequent (TC), Strengthening of the antecedent (S), Contraposition (C), Transitivity (T), Conditional Excluded Middle (CEM), Simplification of Disjunctive Antecedents (SDA), Or-to-If (OI), Import-Export (IE), Conditional Negation (CN) (the latter being the equivalence between $\neg(A \rightarrow C)$ and $A \rightarrow \neg C$), Modus Ponens (MP) and Modus Tollens (MT). "Plural" refers to the plural version of Stalnaker's analysis (no Uniqueness Assumption), and "Strict" to the basic strict conditional analysis. For the 3-valued analysis, the rightmost column assumes De Finetti's scheme for the conditional under the S3 or symmetric version of Kleene consequence (preservation of value 1 from premises to conclusion, and of value 0 from conclusion to premises). For the Strict analysis, Stalnaker and Lewis's analyses, and the Plural analysis, the Table assumes the the actual world is always among the accessible worlds (reflexivity) and also the assumption of Centering (if the actual world is an A-world, then it is one of the closest A-worlds or the closest). For McGee's analysis, we refer to his modification of Stalnaker's semantics, and assume schemata to obey the syntactic proviso of the semantics (conditionals can only have factual antecedents). The parallel between the predictions of McGee's semantics and De Finetti's scheme (under S3) may suggest that the two semantics are equivalent throughout on their common language. This is not the case, however. For example, as pointed out in section 1.4, the embedded conditional $(A \vee C) > (\neg A > C)$ is McGee-valid (assuming, again, factual antecedents), but it is not valid for De Finetti's conditional under the S3 scheme (for instance when A is true).

	Material	Strict	Stalnaker	Plural	Lewis	McGee	De Finetti
FA	+	-	-	-	-	-	-
TC	+	-	-	-	-	-	-
S	+	+	-	-	-	-	-
C	+	+	-	-	-	-	-
T	+	+	-	-	-	-	-
CEM	+	-	+	-	-	-	-
SDA	+	+	-	-	-	-	-
OI	+	-	-	-	-	-	-
IE	+	-	-	-	-	+	+
CN	-	-	+	-	-	+	+
MP	+	+	+	+	+	-	-
MT	+	+	+	+	+	-	-

Table 1.2 *Overview of the frameworks / inference schemata discussed*

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