Introduction to the Logic of Conditionals
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Lecture 4. Triviality Results and their implications

F.P. Ramsey

the Ramsey Test

- summary: we have seen two main ways of elaborating the so-called Ramsey Test, one in the belief revision framework (RT_f), the other in Bayesian probability (Adams Thesis)
- all this has to do with the **epistemology of conditionals**, even if we saw how it *could* have a strong impact on the **semantics of conditionals**
- it's time now to see how the Ramsey Test interacts with (i) other fundamental tenets of epistemology and (ii) semantics
- e.g.: ideally, from a semantic point of view, one would like truth-conditions for $\Rightarrow$ s.t. (AT) derives from these truth-conditions plus basic principles of probabilities ($p([\![A \Rightarrow C]\!]) = p([\![C]\!]|[\![A]\!])$)

triviality results

- at this point enter the “Bombshell”(s): an impressive sequence of **triviality results** intended to show basically that, on pain of triviality, the Ramsey Test cannot live peacefully with basic tenets of epistemology and/or semantics
- Menu:

(1) we will expose these triviality results

(2) we will then discuss the main *reactions* to these results (formal moves as well as lessons drawn from these) - mainly for the probabilistic results

Triviality Results

Lewis Triviality Results (1976)

- the connective $\Rightarrow$ is **probability conditional** for a class **P** of probability functions iff for every $P \in \mathbf{P}$ and formulas A, C with $P(A) > 0$,

$$\text{(AT) } P(A \Rightarrow C) = P(C|A)$$

- the connective $\Rightarrow$ is a **universal probability conditional** iff (AT) holds for every probability function
- question: for a suitable (not necessarily universal) **P**, does there exist a probability conditional ?

the factorization hypothesis

- the main assumption of LTR is the Factorization Hypothesis (FH):

$$P(A \Rightarrow C|B) = P(C|AB), \text{ if } P(AB) > 0$$

- there are two different lines of argument supporting (FH)

factorization hypothesis and conditionalization

(i) **(FH) and conditionalization**: suppose that (AT) holds for every $P \in \mathbf{P}$ and that $\mathbf{P}$ is closed by conditionalization. Then (FH) holds for every $P \in \mathbf{P}$.

(1) $P(A \Rightarrow C|B) = P_B(A \Rightarrow C)$ (by closure)
(2) $P(A \Rightarrow C|B) = P_B(C|A)$ (by (AT) applied to P_B)
(3) $P(A \Rightarrow C|B) = P_B(C \wedge A)/P_B(A)$ (by Ratio Formula)
(4) $P(A \Rightarrow C|B) = P(C \wedge A \wedge B)/P(A \wedge B)$
(5) $P(A \Rightarrow C|B) = P(C|AB)$

factorization hypothesis and Import-Export

- **(FH) and the Import-Export Law (IE)**: suppose that (AT) holds for P. Then (FH) is equivalent to the probabilistic version of Import-Export

  $$\text{(PIE) } P(B \Rightarrow (A \Rightarrow C)) = P(AB \Rightarrow C), \text{ si } P(AB) > 0$$

  Proof: Supp. (PIE).

  (1) $P(A \Rightarrow C|B) = P(B \Rightarrow (A \Rightarrow C))$ (by (AT))
  (2) $P(A \Rightarrow C|B) = P(AB \Rightarrow C)$ (if $P(AB) > 0$ by (PIE))
  (3) $P(A \Rightarrow C|B) = P(C|AB)$ (if $P(AB) > 0$ by (AT))
  The other direction is analog. ♠

Lewis First Triviality Result

- **Theorem (LTR1):** Suppose that (AT) and (HF) holds for a class **P**. For any $P \in \mathbf{P}$, if $P(A \wedge C) > 0$ and $P(A \wedge \neg C) > 0$, then $P(C|A) = P(C)$

- *Proof:*
  (1) $P(A \Rightarrow C) = P(C|A)$ (AT)
  (2) $P(A \Rightarrow C|C) = P(C|A \wedge C) = 1$ (FH)
  (3) $P(A \Rightarrow C|\neg C) = P(C|A \wedge \neg C) = 0$ (FH)
  (4)$P(A \Rightarrow C) = P(A \Rightarrow C|C) \cdot P(C)$
  $+P(A \Rightarrow C|\neg C) \cdot P(\neg C)$ (expansion by case)
  (5) $P(C|A) = 1 \cdot P(C) + 0 \cdot P(\neg C) = P(C)$ (by (1)-(4)) ♠

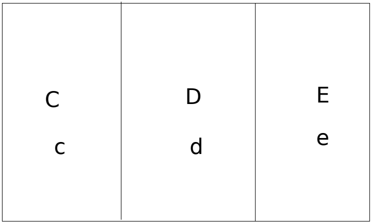
example

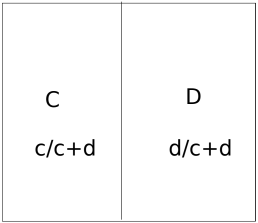
- the Result says that any two sentences A and C are probabilistically independent. But this excludes almost every probability function !
- example: the throw of a fair dice described by P. Let
  A="an even number comes up"
  C="the 6 comes up"
- the assumptions are satisfied : $P(A \wedge C) = \frac{1}{6}$ and $P(A \wedge \neg C) = \frac{1}{3}$
  But $P(C|A) = \frac{1}{3}$ whereas $P(C) = \frac{1}{6}$
- a probability function as simple and natural as P is therefore excluded from **P** !

Lewis Second Triviality Result

- let's generalize from this example: suppose that C, D, E are three pairwise incompatible formulas, that each of them is possible given the semantics. If P assigns a positive weight to each of them, then $P(C|(C \vee D) \neq P(C)$ (C receives at least some of the weight of E)
- a **trivial language** is a language that does not contain such formulas.

non-trivial language





Lewis Second Triviality Result

- LTR1 implies that "any language having a probability conditional is a trivial language"
- **Theorem (LTR2)** If (AT) and (FH) hold for *P*, then *P* assigns non-zero probabilities to at most two of any set of pairwise incompatible formulas
  (a trivial language is a sufficient but no necessary condition for this condition)

Lewis Third Triviality Result

- **Theorem (LTR3)** If (AT) and (FH) holds for P, then P takes at most four values.

  Proof: if P has more than four values, then there exists C and D s.t. $P(C) = x$ and $P(D) = y$ with $x + y \neq 1$. Hence $x \neq y$ and $(1 - x) \neq (1 - y)$. So $P(.)$ has at least 5 values. Suppose w.l.o.g. that $x + y < 1$. If $E = \neg C \wedge \neg D$, then $P(E) > 0$. H is (pairwise) incompatible with C and D.

  1. if C and D are incompatible, then C, D and E are all pairwise incompatible and non-zero weighted
  2. if $C \models D$, then C, $D \wedge \neg C$ and E are all pairwise incompatible and non-zero weighted (the same reasoning holds if $D \models C$)
  3. if C and D are not incompatible without one being the consequence of the other, then $C \wedge \neg D$, $D \wedge \neg C$ and E are all pairwise incompatible and non-zero weighted

what about the Stalnaker conditional?

- since $A > C$ is given truth-conditions (see Lecture 1), you may in principle perfectly deal with probabilities of Stalnaker conditional
- but if given a selection function f on W, $P(A > C)$ *is not* in general $P(C|A)$. A revision process called **imaging** by D.Lewis corresponds to $>$ i.e.

$$P^I_A(C) = P(A > C).$$

imaging

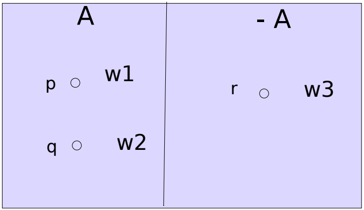
- the imaging rule is simple: the weight of a world w' excluded by the information that ϕ is wholly transferred to $f(\phi, w')$:

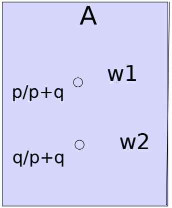
(i) if $w \notin [\![\phi]\!]$, $P^I_\phi(\{w\}) = 0$

(ii) if $w \in [\![\phi]\!]$, w keeps its initial weight *and* receives the weights of every world w' s.t. (i) $w' \notin [\![\phi]\!]$ and (ii) $w = f(\phi, w')$

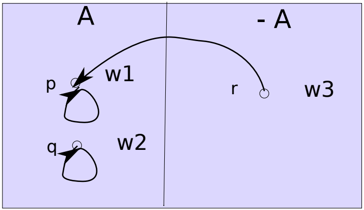
$$P^I_\phi(\{w\}) = \sum_{\{w' \in W : f(\phi, w') = w\}} P(\{w'\})$$

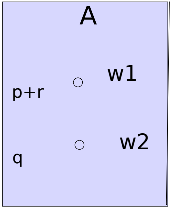
conditionalization on *A*





imaging on *A*





from Lewis to Gärdenfors

- to sum up: when we start from Adams construal of the Ramsey Test, we arrive at devastating triviality results
- maybe this construal is not good after all, and the issue disappears when we look at another framework...

Gärdenfors Triviality Result (1986)

- reminder: in a belief revision framework, the Ramsey Test means

$$(\mathrm{RT}_f)\ A \Rightarrow C \in K \text{ iff } C \in K * A$$

- Gärdenfors's assumptions:

(K*2) $\phi \in K * \phi$ **(Success)**

(K*5w) if $K \neq K_\perp$ and $K * \phi = K_\perp$, then $\models \neg\phi$ **(Consistency)**

(K*P) if $\neg\phi \notin K$ and $\psi \in K$, then $\psi \in K * \phi$ **(Gärdenfors Preservation Condition)**
idea: don't give up beliefs unnecessarily !

Gärdenfors Triviality Result

- **Gärdenfors Triviality Result**: there is no non-trivial belief revision model (b.r.m.) that satisfies (RT_f) and (K*2), (K*5w) and (K*P).
- a b.r.m. is *non-trivial* iff there is at least 3 pairwise incompatible formulas and a belief set K which is consistent with these formulas ($\neg\phi_i \notin K$) (very close to Lewis definition)
- surprisingly, the proof doesn't rely directly on (RT_f) but on a consequence of (RT_f), **Monotonicity** (which doesn't involve conditionals):

  (K*M) $\forall K, K' \in \mathbf{K}$ and ϕ, if $K \subseteq K'$, then $K * \phi \subseteq K' * \phi$

proof

Bradley's Triviality Result

- let's come back to probabilities. Hajek & Hall (1994) (available upon request) provides an excellent overview of these results for 1976-1994
- two recent results by R. Bradley (2000, 2006) that rely on assumptions much weaker than (AT) (and (FH))
- **Bradley Preservation Condition**: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ but $P(C) = 0$, then $P(A \Rightarrow C) = 0$

  [if A is supposed to be possible but C impossible, then the conditional "If A, then C" is supposed to be impossible]

Bradley Preservation Condition

- **Bradley Preservation Condition**: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ but $P(C) = 0$, then $P(A \Rightarrow C) = 0$

  [if A is supposed to be possible but C impossible, then the conditional "If A, then C" is supposed to be impossible]
- example: the following epistemic situation violates Bradley Preservation Condition:

  (1) It might be the case that we go to the beach.

  (2) It is certain that we won't go swimming.

  (3) It might be the case that if we go to the beach, we will go swimming.
- Preservation Condition is implied by (AT) - but not conversely

Bradley Triviality Result

- relative to a consequence relation $\models$, a set of formulas $\mathcal{L}$ is **non-trivial** if it contains two factual sentences A, B and a simple conditional $A \Rightarrow B$ s.t. neither A nor $A \Rightarrow B$ implies B
- **Bradley Triviality Result 1**: if the Preservation Condition holds for every probability function on $\mathcal{L}$, then $\mathcal{L}$ is trivial.
- BTR1 relies on a simple property of probability distribution on (partially ordered) Boolean algebras: if $\neg Y \leq Z$ and $\neg X \leq Z$, there exists P on $(\Omega, \leq)$ s.t. $P(Y) > 0$, $P(Z) = P(X) = 0$.

Bradley Conservation Condition

- another consequence of (AT):

  Bradley Conservation Condition: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ and $P(C) = 1$, then $P(A \Rightarrow C) = 1$

  [if A is supposed to be possible and C certain, then the conditional “If A, then C” is certain]

- **Bradley Triviality Result 2 (2006)**: Assume that Preservation & Conservation Conditions hold for every $P \in \mathbf{P}$ and that $\mathbf{P}$ is closed by conditionalization. If $P(A|C), P(A|\neg C) > 0$, then $P(A \Rightarrow C) = P(C)$

Bradley Triviality Result

- *Proof:*
  (1) $P(A \Rightarrow C) = P((A \Rightarrow C)|C) \cdot P(C)$
  $+P((A \Rightarrow C)|\neg C) \cdot P(\neg C)$ (expansion by case)
  (2) $P(C|C) = 1$ and $P(C|\neg C) = 0$ (standard laws of probability)
  (3) $P((A \Rightarrow C)|C) = 1$ (by Conservation and (2) since $P(A|C) > 0$
  (4) $P((A \Rightarrow C)|\neg C) = 0$ (by Preservation and (2) since $P(A|\neg C > 0$
  (5) $P(A \Rightarrow C) = P(C)$ (by (1), (3) and (4)) ♠

Lord Russell has been murdered

scenario: Lord Russell been murdered. Three suspects: the butler, the cook and the gardener. The butler did it probably (a motive and no alibi). The cook has no known alibi but no motive. The gardener has an alibi and no motive.

1. $P(\overline{C}) = 2/3$ [probably not the cook]
2. $P(\overline{B} \Rightarrow C) = 2/3$ [probably if not butler then cook]
3. $P(\overline{B} \Rightarrow G) = 1/3$ [improbable if not butler, gardener]
4. $P(\overline{B} \Rightarrow G|C) = 0$ [impossible, given cook, if not butler, gardener]
5. $P(\overline{B} \Rightarrow G|\overline{C}) = 1$ [certain, given not cook, if not butler, gardener]

Lord Russell has been murdered

- the probabilities:
  1. $P(\overline{C}) = 2/3$ [probably not the cook]
  2. $P(\overline{B} \Rightarrow C) = 2/3$ [probably if not butler then cook]
  3. $P(\overline{B} \Rightarrow G) = 1/3$ [improbable if not butler, gardener]
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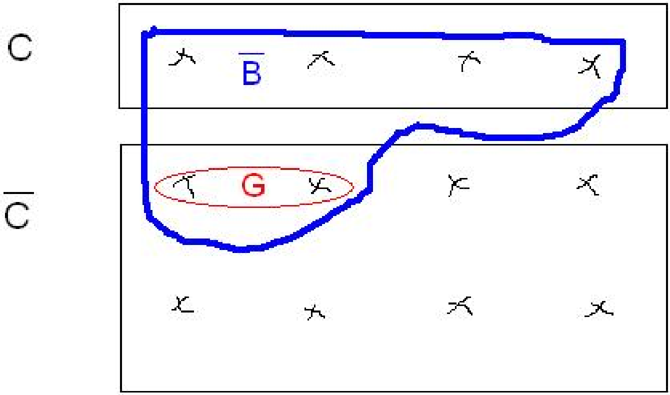
Lord Russell has been murdered

Yet intuitively: conditions 1-5 are jointly satisfiable.



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Then

- $P(\overline{B} \Rightarrow G) = P(\overline{B} \Rightarrow G|C)P(C) + P(\overline{B} \Rightarrow G|\overline{C})P(\overline{C})$
- $P(\overline{B} \Rightarrow G) = 0 \cdot 1/3 + 1 \cdot 2/3 = 2/3$
- $1/3 = 2/3$: contradiction.

2 Ways Out

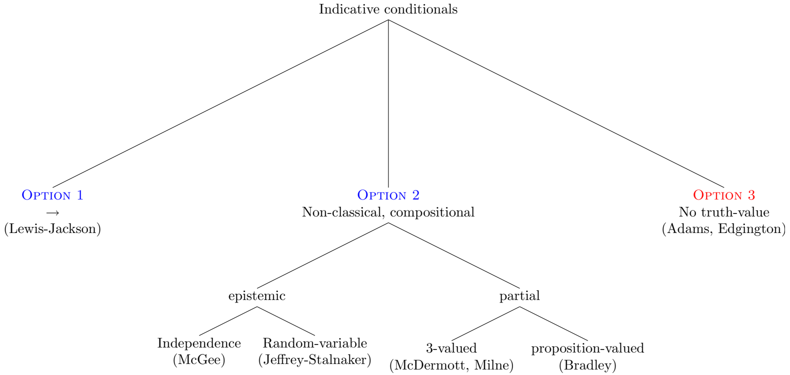
triviality: is there a way out?

- *if* (AT) is true *and* (indicative) conditionals have a classical truth-conditional semantics, then Lewis's assumptions are plausible and we end up with triviality

  "...we cannot have our cake and eat it, and we have to choose. If we accept the conditional probability thesis we have to give up truth-conditionality, and if we accept truth-conditionality, we have to give up the conditional probability thesis" (Adams, 1998)

triviality: which ways out?

- **OPTION 1:** give up (AT) and stick to a classical semantics for $\Rightarrow$ (Lewis, Jackson)
- **OPTION 2:** stick to (AT) but provide a non-classical semantics (Bradley, McDermott, Stalnaker & Jeffrey, McGee)
- **OPTION 3:** stick to (AT) and reject truth-conditions for $\Rightarrow$ (Bennett, Edgington, Appiah, Levi)


Indicative conditionals
OPTION 1
→
(Lewis-Jackson)
OPTION 2
Non-classical, compositional
OPTION 3
No truth-value
(Adams, Edgington)
epistemic
partial
Independence
(McGee)
Random-variable
(Jeffrey-Stalnaker)
3-valued
(McDermott, Milne)
proposition-valued
(Bradley)

OPTION 1: good bye Adams

- **OPTION 1:** reject (AT) and stick to classical semantics for $\Rightarrow$
- some give up (AT) (or the Ramsey Test in general): position endorsed by Gärdenfors for (RT_f) and by Jackson (2006, MS.) for (AT): “Our usage of the indicative conditional construction is governed by a mistaken intuition”
- main elaboration of OPTION 1 due to Lewis (1976) and Jackson (1979): truth-functional semantics for $\Rightarrow$ + substitute for (AT) = the truth-functional view

the truth-functional view: Lewis & Jackson

- here are the main tenets of the truth-functional view (or "New Horseshoe Theory" - Lycan):
- indicative conditionals have truth-conditions
- the truth-conditions of indicative conditionals are those of the material conditional
- the probability of **truth** of an indicative conditional is the probability of the corresponding material conditional
- the **assertability** of an indicative conditional goes (nonetheless) by conditional probability (substitute for (AT))
- the divergence between assertability and probability of truth is explained by some pragmatic principle

conditional probability and robustness

- question : how to derive "assertability goes by conditional probability" a from pragmatic principle ?
- Jackson's theory (1979): **robustness**: when someone asserts "If A, then C", he believes strongly that $A \rightarrow C$ *and* indicates that his or her belief is robust with respect to the antecedent i.e. his or her belief in the conditional would still be strong were he or she learns that the antecedent is true
- this theory predicts that the assertability of $(A \Rightarrow C)$ depends on $P(A \rightarrow C|A)$
  $P(A \rightarrow C|A) = P(\neg A \vee C|A) = P(AC|A) = P(C|A)$

why robustness ?

- what is the point of robustness ?
- Jackson's explanation: robustness w.r.t. the antecedent guarantees the use of Modus Ponens.
- in general, if you believe $(A \rightarrow C)$, don't know about A and C and are interested by C, you will inquire whether A. But evidence for A can be evidence against $(A \rightarrow C)$. If this is so, you won't be in a position to infer C by MP.
- not so if robustness w.r.t. the antecedent

objections to Lewis-Jackson

- objection 1 (e.g. Bradley 2002): it is not only the degree to which we are disposed to assert a conditional that equates the conditional probability but the degree to which we believe it
- objection 2: embeddings
  (i) embbedings of material conditionals are problematic:

  (4) If John is in Paris, he is in France

  (5) Either if Jones is in Paris, he is in Turkey, or if John is in Istanbul, he is in France

  (ii) how to explain non-semantically the trouble with this kind of inference ? how to extend the basic explanation beyond simple conditionals?

objections to Lewis-Jackson, cont.

- objection 3 (Edgington): the analogy between $\Rightarrow$ vs. $\rightarrow$ and “but” vs. “and” often invoked by Jackson to motivate his theory is not satisfactory:
  we would say that

  (6) Paul is French but smart

  is probably true but inappropriate ; not so for

  (7) If Jacques Chirac is elected President for the third time, he will double income tax

OPTION 2: let's have the cake and eat it !

- **OPTION 2:** stick to (AT) but provide a non-classical semantics
- two main alternatives:

(a) either a belief-independent non-classical semantics (coined thereafter **partial semantics** for reasons that will become clear), or

(b) a belief-dependent non-classical semantics (coined **epistemic semantics**)

McGee Logic (MGL)

- McGee (1989) "Conditional Probabilities and Compounds of Conditionals" is the main existing attempt to develop Adams's work in the face of Lewis's Triviality Results
- McGee allows the embedding of conditionals in the consequents of conditionals: conditionals have the general form

$$(A \Rightarrow \phi)$$

where A is a factual formula and there is no restriction on ϕ. Let $\mathcal{L}_2^{\Rightarrow}$ denote MGL language:
- $F := p \mid \neg F \mid F \vee F$ (= $\mathcal{L}_0$)
- $\phi := F \mid F \Rightarrow \phi$
($\mathcal{L}_0 \subseteq \mathcal{L}_1^{\Rightarrow} \subseteq \mathcal{L}_2^{\Rightarrow} \subseteq \mathcal{L}_3^{\Rightarrow}$)

McGee Logic (MGL)

- McGee's main assumptions:

  ✓ the standard axioms of probability functions (P1)-(P4)
  ✓ the Import-Export Law ($P(B \Rightarrow (A \Rightarrow C)) = P(AB \Rightarrow C)$, if $P(AB) > 0$)
  ✓ Adams Thesis restricted to *simple conditionals*

- instead of the full Adams Thesis, McGee requires a principle called the **Independence Principle**

McGee Logic (MGL)

- (simple) **Independence Principle** : if A and C are classically incompatible and $P(A) > 0$, then

$$P(C \wedge (A \Rightarrow B)) = P(C).P(A \Rightarrow B)$$

  which implies that the probability of $(A \Rightarrow B)$ should not change on the assumption that the antecedent is false

  why?

- example:
  $P(ODD \wedge (EVEN \Rightarrow SIX) = P(ODD) \cdot P(EVEN \Rightarrow SIX)$
- under the assumption that $P(A \wedge (A \Rightarrow B)) = P(A \wedge B)$, it implies Adams Thesis for simple conditionals proof

sketch of the machinery

- we saw in Lecture 3 that (AT) allows us to extend P defined on $\mathcal{L}_0$ to simple conditionals ($\mathcal{L}_1^{\Rightarrow}$)
- the Independence Principle (+ Modus Ponens for factual formulas) allows us to extend P on $\mathcal{L}_0$ to Boolean compounds of factual formulas and simple conditionals
- Import-Export allows us to extend P to right-nested conditionals hence to $\mathcal{L}_2^{\Rightarrow}$

first example: $P(B \wedge (A \Rightarrow C))$

- what is $P(B \wedge (A \Rightarrow C))$?
- intuitive idea:
  - when $\neg B$, the whole is false
  - when ABC, the whole is true
  - when $AB\neg C$, the whole is false
  - when $\neg AB$, the value is $P(A \Rightarrow C)$
- $P(B \wedge (A \Rightarrow C)) = P(ABC) + P(\neg ABC) \cdot P(A \Rightarrow C)$

first example: $P((A \Rightarrow B) \wedge (C \Rightarrow D))$

- what is $P((A \Rightarrow B) \wedge (C \Rightarrow D))$?
- intuitive idea:

  • when $ABCD$, the whole is true
  • when $AB\neg C$, the value is $P(C \Rightarrow D)$
  • when $\neg ACD$, the value is $P(A \Rightarrow B)$

  and all is normalized by $P(A \vee C)$

- $P((A \Rightarrow B) \wedge (C \Rightarrow D)) = 1/P(A \vee C)\cdot$
  $[P(ABCD)$
  $+P(\neg ACD) \cdot P(A \Rightarrow B)$
  $+(P(AB\neg C) \cdot P(C \Rightarrow D)]$

McGee and Lewis

- let's have a look at how McGee deals with Lewis trivialization:
  (1) $P(A \Rightarrow C) = P(C|A)$ (AT)
  (2) $P(A \Rightarrow C|C) = P(C|A \wedge C) = 1$ (FH)
  (3) $P(A \Rightarrow C|\neg C) = P(C|A \wedge \neg C) = 0$ (FH)
  (4)$P(A \Rightarrow C) = P(A \Rightarrow C|C) \cdot P(C)$
  $+P(A \Rightarrow C|\neg C) \cdot P(\neg C)$ (expansion by case)
  (5) $P(C|A) = 1 \cdot P(C) + 0 \cdot P(\neg C) = P(C)$ (by (1)-(4)) ♠
- (FH) is not valid ; since (PIE) is, this implies that in general $P(B \Rightarrow (A \Rightarrow C)) \neq P(A \Rightarrow C|B)$

MGL and Modus Ponens

- a surprising feature of MGL: Modus Ponens is not valid ! (it is valid only for simple conditionals)
- McGee's scenario (1985):
  3 candidates in 1980 presidential campaign, Reagan (Republican, ahead in the polls), Carter (Democrat, second) and Anderson (Republican, distant third)
  R : "Reagan will win the election"
  A : "Anderson will win the election"
- the problematic inference:
  $((R \vee A) \Rightarrow (\neg R \Rightarrow A))$
  $(R \vee A)$
  $\therefore (\neg R \Rightarrow A)$

Stalnaker & Jeffrey (1994)

- Stalnaker & Jeffrey (1994) propose *"an embedding of Adams' treatment of the simplest conditionals in a more permissive or comprehensive framework allowing arbitrary embeddings of conditional sentences within each other and within truth functional compounds."*
- idea: the semantic value of a factual formula A can be seen as a function from possible worlds to truth value and therefore as a (degenerate) random variable δ_A
- the probability of a factual formula can be seen as the expectation of its random variable: $P(A) = \mathbf{E}(\delta_A)$ (weighted average of the values of δ_A on possible worlds)

Expectation-Based Adams Thesis

- you can see in general the semantic value of a sentence ϕ as a random variable δ_ϕ
- new formulation of Adams Thesis = **Expectation-Based Adams Thesis (EBAT)** :

$$\mathbf{E}(\delta_{A \Rightarrow \phi}) = \mathbf{E}(\delta_\phi | \{w : \delta_A(w) = 1\})$$

an example from Edgington (2006)

- (EBAT) : $\mathbf{E}(\delta_{A\Rightarrow\phi}) = \mathbf{E}(\delta_\phi | \{w : \delta_A(w) = 1\})$
- scenario: 50% of the balls are red (R), 80% of the red balls have a black spot (B)
- the semantic value of $R \Rightarrow B$ is a random variable $\delta_{R\Rightarrow B}$ defined on W:

✓ $\delta_{R\Rightarrow B}(w) = 1$ if $w \in [\![R \wedge B]\!]$
✓ $\delta_{R\Rightarrow B}(w) = 0$ if $w \in [\![R \wedge \neg B]\!]$
✓ $\delta_{R\Rightarrow B}(w) = P([\![B]\!]|[\![R]\!]) = 8/10$ if $w \in [\![\neg R]\!]$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![R \wedge B]\!]) \cdot 1 + P([\![R \wedge \neg B]\!]) \cdot 0 +$
$P([\![\neg R]\!]) \cdot 8/10$
$\mathbf{E}(\delta_{R\Rightarrow B}) = 4/10 + 5/10 \cdot 8/10 = 8/10 = P([\![B]\!]|[\![R]\!])$!

- this is not a coincidence !

$$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![R \wedge B]\!]) \cdot 1 + P([\![R \wedge \neg B]\!]) \cdot 0 + (1 - P([\![R \wedge B]\!]) - P([\![R \wedge \neg B]\!])) \cdot P([\![B]\!] | [\![R]\!])$$

$$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!] | [\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot P([\![B]\!] | [\![R]\!] - P([\![R \wedge \neg B]\!]) \cdot P([\![B]\!] | [\![R]\!])$$

$$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!] | [\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot [P([\![R \wedge B]\!]) + P([\![R \wedge \neg B]\!]) / P([[\![R]\!])$$

$$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!] | [\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot 1$$

$$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!] | [\![R]\!])$$

random variable semantics

- let's come back on the definition of $\delta_{R \Rightarrow B}$. Crucial case: the $\neg R$-worlds where $\delta_{R \Rightarrow B}(w) = P(\llbracket B \rrbracket | \llbracket R \rrbracket)$.
- as Edgington (2006) puts it, the $\llbracket \neg R \rrbracket$-worlds are not divided in $R \Rightarrow B$-worlds and $\neg R \Rightarrow B$-worlds but in these worlds $R \Rightarrow B$ is so to speak "true to degree $P(\llbracket B \rrbracket | \llbracket R \rrbracket)$"

A	C	A → C
1	1	1
1	0	0
0	1	p(C|A)
0	0	p(C|A)

Figure: from Jeffrey (1991)

random variable semantics, cont.

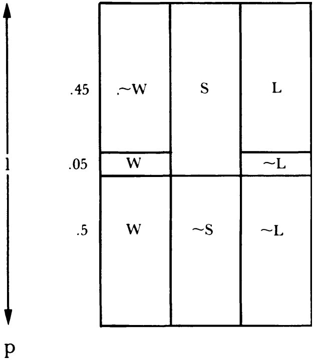
- note that the “semantic value” of the conditional is uniform in the worlds where the antecedent is false. This is typically not so with counterfactuals.
- the “semantic value” of conditionals depends on the underlying partial beliefs, therefore it is an **epistemic semantics**
- if David and Paul have not the same partial beliefs, the “semantic value” of the conditionals they express may differ in the very same possible world (see below)

objections to epistemic semantics

- objection 1: belief-dependency
  "...it goes against a strong intuition that we don't intend to just express our beliefs when we assert conditionals, and that we intend to say something about the way that the world is." (Bradley 2002)
  an example
- objection 2: embbedings
  the main value of epistemic semantics w.r.t. Adams Logic lies in the treatment of embedded conditionals. But according to Edgington, the prediction s are bad

first example on embedding

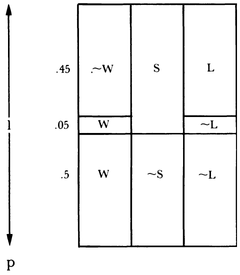
- *S*: David will strike the match *L* : the match will light *W*: the match is wet



- how likely "If the match is wet, if David strike it, it will light" $(W \Rightarrow (S \Rightarrow L))$?

first example on embedding

- intuitive answer: 0
- random variable semantics answer: $P(S \Rightarrow L|W)$ which can be calculated as 0.82 !
- McGee system answer: $P(W \wedge S) \Rightarrow L = 0$



second example on embedding

- another example: when A and C are incompatible, in both theories $P(A \Rightarrow B) \wedge (C \Rightarrow D) = P(A \Rightarrow B) \cdot (C \Rightarrow D)$
- let's consider a fair coin which is tossed (independently) two times

(8) It will land heads at the first toss (H_1)

(9) It will land heads at the second toss (H_2)

(10) If it lands heads at the first toss, it will land heads at the second toss ($H_1 \Rightarrow H_2$)

(11) If it does not land heads at the first toss, it will land heads at the second toss ($\neg H_1 \Rightarrow H_2$)

- intuitive probability: 1/2 (?); but the theories delivers $1/2 \cdot 1/2 = 1/4$!

partial semantics for conditionals

- second main family of proposal inside OPTION 2 = **partial semantics**
- simplest formulation of the idea (De Finetti 1937, Von Wright 1957, McDermott 1996, Milne, 1997): $A \Rightarrow C$

  (a) is true when AC,
  (b) false when $A\neg C$, and
  (c) neither true nor false when A is false
- this three-valued semantics shares some structural features with the random variable semantics: $A \Rightarrow C$ has the value of C when A is true, and an uniform value when A is not true

conditionals and truth-value gaps

- here are truth-tables for $\Rightarrow$ and two pairs of conjunction/disjunction ($\wedge/\vee$) and ($\cap, \cup$) (from McDermott (1996)):

ϕ	ψ	$\phi \to \psi$	$\sim\phi$	$\phi \wedge \psi$	$\phi \cap \psi$	$\phi \vee \psi$	$\phi \cup \psi$
T	T	T	F	T	T	T	T
T	F	F	F	F	F	T	T
T	X	X	F	X	T	T	T
F	T	X	T	F	F	T	T
F	F	X	T	F	F	F	F
F	X	X	T	F	F	X	F
X	T	X	X	X	T	T	T
X	F	X	X	F	F	X	F
X	X	X	X	X	X	X	X

partial semantics for conditionals, cont.

- this semantics doesn't deliver (AT) directly since $P([\![A \Rightarrow C]\!]) = P([\![A \wedge C]\!])$
- but you can see the proposition expressed by a sentence ϕ no longer as $[\![\phi]\!]$ but as a pair $([\![\phi]\!]_1, [\![\phi]\!]_0)$ where $[\![\phi]\!]_1 = \{w : v_w(\phi) = 1\}$ and $[\![\phi]\!]_0 = \{w : v_w(\phi) = 0\}$
- you can then define a notion of credence c as the probability that a sentence is true given that is has a truth-value:

$$c(\phi) = P([\![\phi]\!]_1 | [\![\phi]\!]_1 \cup [\![\phi]\!]_0)$$

- if $c(\phi) = P([\![\phi]\!]_1 | [\![\phi]\!]_1 \cup [\![\phi]\!]_0)$, then it follows immediately that

  • for any $A \in \mathcal{L}_0$, $C(A) = P(A)$
  • for any $A \Rightarrow$, $C(A \Rightarrow C) = P(C|A)$ (Adams Thesis for simple conditionals)

- the prediction of the semantics depends on the exact behavior of (ex-)Boolean connective. If one endorses the strong conjunction $\wedge$ (as apparently De Finetti 1937 did),

  • $A \Rightarrow (B \Rightarrow C) \equiv_3 (A \wedge B) \Rightarrow C$
  • $\neg(A \Rightarrow C) \equiv_3 (A \Rightarrow \neg C)$

objections

- objection 1: the strong conjunction (Edgington, Bradley)
  - $\wedge$ is appealing but a *partitioning sentence* $(A \Rightarrow B) \wedge (\neg A \Rightarrow C)$ is never true according to the semantics (at least one of the conditional has no truth-value) !
  - it follows that $c((A \Rightarrow B) \wedge (\neg A \Rightarrow C) = 0$!
- objection 2: the weak conjunction (Bradley 2002)
  - $\neg((A \Rightarrow B) \cap (\neg A \Rightarrow C) \equiv_T (A \Rightarrow \neg B) \cap (\neg A \Rightarrow \neg C)$

(12) It is not true that if you go left you will get to the shops and if you go right you will get to the post-office

(13) If you go left you won't get to the shops and if you go right you won't get to the post-office

objections, cont.

- last issue: we obtain Adams Thesis by equating our belief attitude towards $A \Rightarrow C$ with the credence function $c(.)$; but why should it be so ?
- Edgington (1995a) : *"The "true, false, neither" classification does not yield an interesting 3-valued logic or a promising treatment of compounds of conditionals ..."*

Bradley's semantics

- Bradley (2002) proposes a more sophisticated version of partial semantics according to which "conditionals unlike factual sentences determine propositions only in those contexts in which their antecedents are true"
- $A \Rightarrow C$ expresses the proposition $[\![AC]\!]$ in worlds where A is true and no proposition otherwise
- a sentence ϕ is true at w if the proposition expressed at w is true at w, false if the proposition expressed is false and neither if it expresses no proposition

No Truth Value

OPTION 3: No Truth Value

- let's now turn to OPTION 3 known currently as the **No Truth Value** view and held mainly by philosophers (Edgington, Bennett, Adams, Appiah, Levi)
- *"If we stick by [(AT)], we must not think of conditionals as propositions, as truth bearers...Your degree of belief that B is true, on the supposition that A is true, cannot be consistently and systematically equated to your degree of belief that something is true, simpliciter."* (Edgington 1995)
- according to this view

1. conditional assertions should not be understood as assertions of conditional propositions
2. conditional beliefs should not be understood as beliefs in conditionals

conditional assertion

- *"An affirmation of the form "if p then q" is commonly felt less as an affirmation of a conditional than as a* ***conditional affirmation*** *of the consequent. If, after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent and are ready to acknowledge error if it proves false. Of on the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made."* (Quine, *Methods of Logic*)

conditional speech acts

- speech acts in general (and not only assertions) can be divided in *categorical* and *conditional* speech acts

speech act	categorical	conditional
assertion	You will go to the beach	If it's sunny, you will go to the beach
question	Will you go to the beach ?	If it's sunny, will you go to the beach ?
command	Go to the beach !	If it's sunny, go to the beach !
promise	I will go to the beach	If it is sunny, I will go to the beach

conditional attitudes

- in the same way, propositional attitudes can be divided in categorical and conditional

attitude	categorical	conditional
belief	David believes that Paul will go to the beach	David believes that if it sunny, Paul will go to the beach
desire	David desires that Paul goes to the beach	David desires that if it sunny, Paul goes to the beach
intention	David intends to go to the beach	David intends to go to the beach if it is sunny

conditional desire

- claim: a NTV theory extends more adequately to other conditional attitudes
- example: I desire that if I win the prize (W), you tell Fred straight away (F)
- propositional account: to desire that $W \Rightarrow F$ = to prefer $W \Rightarrow F$ to $\neg(W \Rightarrow F)$

  ✓ $\rightarrow$: I prefer $\neg W \vee (W \wedge F)$ to $W \wedge \neg F$!
  ✓ $>$: I prefer to be in a world whose nearest W-world is a F-world than to be in a world whose nearest W-world is a $\neg F$-world

- NTV account: to desire that $W \Rightarrow F$ = to prefer F to $\neg F$ conditionally on W = to prefer WF to $W \neg F$

Bennett's construal of NTV

- *"...the Adams theorist holds that the conditional nature of an indicative conditional comes from a relation between two of the speaker's subjective probabilities; he sees that such conditionals are not* ***reports*** *of one's subjective probabilities; so he opts for NTV, the view that in asserting $A \Rightarrow C$ a person* ***expresses*** *his high probability for C given A, without actually saying that this probability is high"* (Bennett 2003)
- analogy with so-called expressivist views for the semantics of evaluative sentences: when David says

  (14) Eating animals is wrong

  he expresses his disapproval of eating animals, he does not report that he disapproves eating animals
- why this distinction between reporting and expressing ?

dialogue involving conditional

- dialogue

(15) Z: If Pete called, he won

(16) D: Are you sure ?

(17) Z: Yes, fairly sure

- how do we understand Z's answer ?
  Z: Yes, fairly sure: I saw both hands, and Pete's was the worse
  Z: Yes, fairly sure: I calculated my ratio of subjective probabilities

dialogue involving evaluation

- dialogue

(18) Z: This film was boring

(19) D: Are you sure ?

(20) Z: Yes, fairly sure

- how do we understand Z's answer ?
  Z: "Yes, fairly sure: during the last hour, almost nothing happened."
  Z: "Yes, fairly sure: I told it during the film to my neighbor"

objections to NTV

- objection 1: “linguistic bizarreness” (Lycan): if-constructions would have no truth-conditions whereas very close constructions would have one

(21) I will leave if you leave.

(22) I will leave when you leave.

(23) If and when she submits a paper, we’ll read it within a month

indicatives and subjunctives

- objection 2: parallels between indicatives and subjunctives
- most of those who endorse NTV for indicative conditionals view subjunctives as truth-valued. This makes mysterious the parallels between indicatives and subjunctives
- this is still more mysterious if one endorses the view that future indicatives are semantically similar to subjunctives (Dudman, 1983).

(24) If you dropped that vase [at *t*], your father found out [non truth-valued]

(25) If you drop that vase [said prior to *t*], your father will find out [truth-valued]

embedding

- objection 3: embedding. A truth-conditional account of conditionals provides an account of embedded conditionals (inside boolean or modal operators, conditionals, etc)
- upholders of NTV reply that

  (i) lots of embedding with conditionals are not intuitively intelligible
  (ii) truth-conditional analyses do not deal satisfactorily with compounds conditionals
  (iii) it is sufficient "to deal *ad hoc* with each kind of embedding without treating indicative conditionals as propositions" (Gibbard)

dealing with embeddings

- examples of *ad hoc* treatments of embeddings:

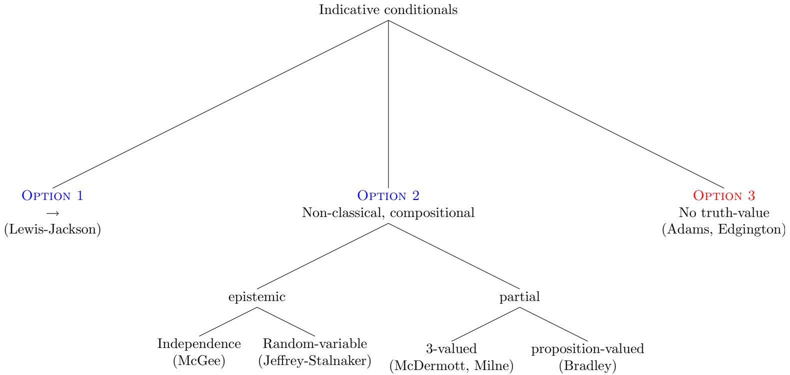
  • assert $\neg(A \Rightarrow C)$ is to assert $\neg C$ conditionally on A
  • assert $B \Rightarrow (A \Rightarrow C)$ is to assert C conditionally on AB
  • assert $(A \Rightarrow B) \wedge (C \rightarrow D)$ is to assert jointly $(A \Rightarrow B)$ and $(C \rightarrow D)$

Summary and Perspectives

summary

- (1) the “Bombshell”(s): lots of triviality results showing that there is a clash between Ramsey Test and basic tenets of semantics and epistemology
- (2) the main reactions to the triviality results

summary


Indicative conditionals
Option 1
→
(Lewis-Jackson)
Option 2
Non-classical, compositional
Option 3
No truth-value
(Adams, Edgington)
epistemic
partial
Independence
(McGee)
Random-variable
(Jeffrey-Stalnaker)
3-valued
(McDermott, Milne)
proposition-valued
(Bradley)

some inclinations and speculations

- we favor OPTION 2, but
  • (1) it is not clear that one has to make sense of arbitrary embeddings of conditionals and
  • (2) it is not clear that if the conditional has a semantics, it behaves as a binary connective (see Lecture 2)
- a view of conditionals as restrictors applied to limited kinds of embeddings ($\mathcal{L}^{\Rightarrow}_{1\frac{1}{2}}$, no conjunction or disjunction forming sentences from conditionals) could be a promising avenue
- could it survive triviality results ?

proof of Gärdenfors Triviality Result

- Lemma: (G1) and (G3) implies (I) if $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$
  Supp. $\psi \in K + \phi$.
  (1) $\phi \rightarrow \psi \in K$ (deduction theorem)
  (2) if $\neg\phi \notin K$, $\phi \rightarrow \psi \in K * \phi$ (by Preservation)
  (3) if $\neg\phi \notin K$, $\psi \in K * \phi$ (by closure and Success)
- Main Proof (by contradiction)
  Let ϕ, ψ, χ formulas pairwise incompatible but consistent with K.
  (1) by (G2) and specific assumptions, $K * \phi * (\psi \vee \chi) \neq K_{\perp}$
  (2) either $\neg\psi \notin K * \phi * (\psi \vee \chi)$ or $\neg\chi \notin K * \phi * (\psi \vee \chi)$ (by G1 and (1))
  (3) suppose w.l.o.g. that $\neg\chi \notin K * \phi * (\psi \vee \chi)$
  (4) $K + (\phi \vee \psi) \subseteq K + \phi \subseteq K * \phi$ (by Lemma)

proof of Gärdenfors, cont.

(5) $(K+(\phi\vee\psi))*(\psi\vee\chi)\subseteq K*\phi*(\psi\vee\chi)$ (by 4 and M)
(6) $\neg\chi\notin(K+(\phi\vee\psi))*(\psi\vee\chi)$ (by (3) and (5))
(7) $\neg(\psi\vee\chi)\notin K+(\phi\vee\psi)$ (spec.assump.+(G2))
(8) $(K+(\phi\vee\psi))+(\psi\vee\chi)\subseteq(K+(\phi\vee\psi))*(\psi\vee\chi)$ (by the inclusion property)
(9) $(K+(\phi\vee\psi))\,(\psi\vee\chi)=K+((\phi\vee\psi)\vee(\psi\vee\chi))=K+\psi$
(10) $(K+\psi\subseteq(K+(\phi\vee\psi))*(\psi\vee\chi)$ (by (8) and (9))
(11) $\neg\chi\in K+\psi$ (by assumption and $\psi\in K+\psi$
(12) $\neg\chi\in(K+(\phi\vee\psi))*(\psi\vee\chi)$. Contradiction

back

Independence Principle

$$P(A \Rightarrow B|C)$$
$$= P(C \wedge (A \Rightarrow B))/P(C)$$
$$= P(C) \cdot P(A \Rightarrow B)/P(C)$$

back

Independence Principle and Adams Thesis

$$
\begin{aligned}
& P(A \wedge C) \\
& = P(A \wedge (A \Rightarrow C) \text{ (probabilistic MP)} \\
& = P(A \Rightarrow C) - P(\neg A \wedge (A \Rightarrow C)) \text{ (by laws of proba.)} \\
& = P(A \Rightarrow C) - (P(\neg A) \cdot P(A \Rightarrow C) \text{ (by IP)} \\
& = (1 - P(\neg A) \cdot P(A \Rightarrow C) \text{ (by laws of proba.)} \\
& = P(A) \cdot P(A \Rightarrow C) \text{ (by laws of proba.)}
\end{aligned}
$$

back

an example of belief-dependency

w	P	$EVEN$	SIX	$EVEN \Rightarrow SIX$
1	1/6	0	0	1/3
2	1/6	1	0	0
3	1/6	0	0	1/3
4	1/6	1	0	0
5	1/6	0	0	1/3
6	1/6	1	1	1

belief-dependency

w	P	$EVEN$	SIX	$EVEN \Rightarrow SIX$
1	0	0	0	1/2
2	0	1	0	0
3	0	0	0	1/2
4	1/3	1	0	0
5	1/3	0	0	1/2
6	1/3	1	1	1

back

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