

Introduction to the Logic of Conditionals

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Lecture 5. Indicative and Subjunctive Conditionals

Where are we?

- ▶ Why care about the triviality results?
- ▶ They show that Adams' thesis, however plausible, cannot hold without restrictions
- ▶ They confirm that if indicative conditionals have truth conditions, then at any rate, these truth conditions are not as straightforward as those of boolean sentences.

The Indicative-Subjunctive distinction

Adams' pair

- (1) If Oswald **did** not kill Kennedy, then someone else **did**.
 - (2) If Oswald **had** not killed Kennedy, someone else **would** have.
- ▶ Different truth-conditions
 - ▶ (1) is true, given what we know about Kennedy's death. (1) is true, under the assumption that Oswald did kill Kennedy, only if one believes in conspiracy theories.

Morphology in English

- ▶ **Indicative conditionals**= **IND** in antecedent, **IND** in consequent.
 - (3) If Mary is rich, then she is happy.
 - (4) If Mary becomes rich, she will be happy.

- ▶ **Subjunctive conditionals**= **SUBJ/PAST** in antecedent, **SUBJ/WOULD** in consequent.
 - (5) If Mary were/was rich, she would be happy.
 - (6) If Mary had been rich, she would have been happy

Why “subjunctive”?

▶ English, **present subjunctive**:

(7) [The site [requires [that java scripts **be** enabled in your browser]]]

(8) *java scripts be enable in your website

▶ English, so-called **past subjunctive**:

(9) [Mary [wishes [she were rich]]].

(10) *Mary were rich.

▶ Iatridou (2000:263): *“By subjunctive I will refer to the morphological paradigm that appears in the complement of verbs of volition and/or command”*

Cross-linguistic variation

Iatridou (2000: 263):

- ▶ *“There are languages in which counterfactual morphology includes subjunctive; that is, subjunctive can be found in the complement of counterfactual wish and the antecedent of counterfactual conditionals (sometimes the consequent as well) (e.g. **German, Icelandic, Spanish, Italian**).*
- ▶ *Some languages do not have a subjunctive at all (**Danish, Dutch**).*
- ▶ *Other languages have a subjunctive but do not use it in counterfactual morphology (**French**, and all of the **Indo-Aryan** languages that have a subjunctive).”*

Salient values

- (11) If (presently) Mary is rich, then she is happy/must be happy. [epistemic, present].
- (12) If (50 years ago) Mary was rich, then she was happy/must have been happy. [epistemic, past]
- (13) If (tomorrow) Mary becomes rich, then she will be happy. [predictive, future]
- (14) If (presently) Mary were rich, she would be happy. [counterfactual, present]
- (15) If (twenty year ago) Mary had been rich, she would have been happy [counterfactual, past]

Epistemic, predictive, counterfactual

Funk 1985, Kaufmann 2005

Different attitudes toward the **antecedent** of the conditional:

- ▶ **Epistemic**: subjective uncertainty about facts that are settled
- ▶ **Predictive**: objective uncertainty (the facts are not yet settled)
- ▶ **Counterfactual**: knowledge to the contrary

Remark: note that present and past indicative conditionals can very well be asserted if the antecedent has just been truthfully revealed to the speaker. In that case: the speaker indicates she takes the assumption on board (she may still doubt about it).

Anderson's example

Anderson 1951

- ▶ Are "subjunctive" and "counterfactual" coextensional?
- ▶ Answer: **No**

(16) If the patient had taken arsenic, he would show exactly the same symptoms that he does in fact show.

- ▶ Indicates that it is possible that the patient took arsenic: hence non-counterfactual.

Mood and Counterfactuality

- ▶ General agreement that “subjunctive” is a **misnomer** (Kaufmann 2005)
- ▶ “Counterfactual” vs. “Non-counterfactual” is more adequate: **semantic** rather than **morphological** distinction
- ▶ The constraint on expressivity seems to be:

Counterfactuality \Rightarrow Subjunctive

or

Indicative \Rightarrow Non-counterfactuality

- ▶ Clearly, however, **Subjunctive \nRightarrow Counterfactuality**.

(NB. Despite this: we may stick to old usage)

One or two kinds of conditionals?

- ▶ **Dualist theories**: e.g. **Lewis** (indicative as material, subjunctive as counterfactual)

I cannot claim to be giving a theory of conditionals in general...there really are two different sorts of conditional; not a single conditional that can appear as indicative or counterfactual depending on the speaker's opinion about the truth of the antecedent (Lewis 1973: 3)

- ▶ **Monist theories**: e.g. **Stalnaker**. Indicative and subjunctive conditionals have identical truth-conditions, but differ in their **presuppositions**.

Why monism appears preferable

- ▶ A unified semantic account, to the extent that it delivers the same predictions as a dualist account, should be preferred (avoid redundancy, more explanatory)
- ▶ Lewis's theory does not work well for indicative conditionals in the first place
- ▶ Stalnaker's theory: a "Y-shaped" theory (account of semantic similarities and pragmatic differences)

Stalnaker's Y-shaped theory

Stalnaker 1975

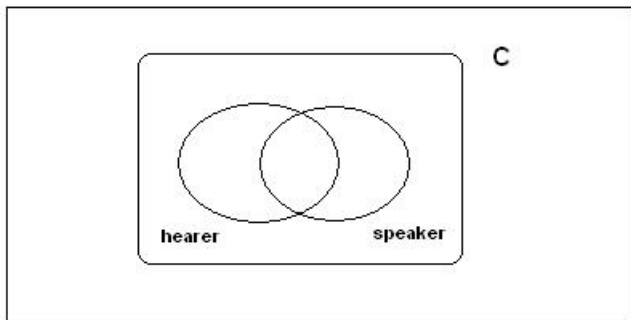
Stalnaker's aim in this paper is two-fold (killing two birds with one stone):

- ▶ Explain how **indicative and subjunctive** conditionals diverge from common truth-conditions
- ▶ Explain how the indicative conditional can get back some desirable properties of the **material conditional**, in particular disjunctive syllogism (see Lecture 1)
- ▶ In both cases: a common pragmatic mechanism

Context Representation

Context set: set of possible worlds compatible with the background information presupposed by the speaker.

- ▶ Larger than the set of belief worlds of the speaker
- ▶ Set of worlds compatible with what the speaker believes to be believed by the hearers.



Making an assumption

- ▶ *when a speaker says "if A", then everything he is presupposing to hold in the actual situation is presupposed to hold in the hypothetical situation in which A is true (Stalnaker 1975)*
- ▶ **Pragmatic constraint on the selection function:** *"if the conditional is evaluated at a world in the context set, then the world selected must, if possible, be within the context set as well"*

Constraint on selection

- ▶ Let $C :=$ context set. Let $f(\phi, C) := \{f(\phi, w); w \in C\}$
- ▶ **Selection constraint:**

$$\text{if } C \cap \phi \neq \emptyset, f(\phi, C) \subseteq C$$

- ▶ *"I would expect that the the pragmatic principle stated above should hold without exception for indicative conditionals" (Stalnaker 1975).*

Possibility of the antecedent

- ▶ *“indicative marking on a conditional if A, B is only felicitous relative to a world w if the context set C contains some A-world”* (Fintel, 1997)
- ▶ $C \cap A \neq \emptyset$ for “if A then B” indicative.
- ▶ Compare with probability of conditionals: $P(A > B)$ defined if $P(A) > 0$

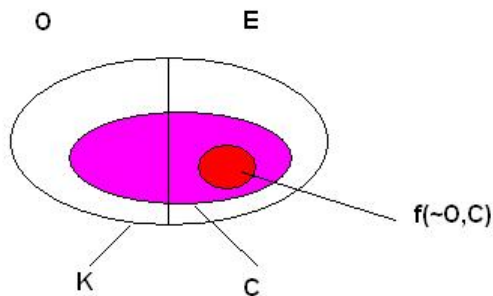
Indicative Oswald

- (17) a. If Oswald did not kill Kennedy, then someone else did.
 b. $\neg O > E$

$K :=$ Kennedy was killed. Assumption: $C \cap \overline{O} \neq \emptyset$

- (i) $K \equiv O \vee E$ (by definition)
- (ii) $C \subseteq K$ (background knowledge)
- (iii) $f(\neg O, C) \subseteq C \subseteq K$ (selection constraint)
- (iv) $f(\neg O, C) \subseteq K \cap \neg O$ (cl1)
- (v) hence, $f(\neg O, C) \subseteq E$. (from iv and i)
- (vi) ie, $w \models \neg O > E$ (def, $w \in C$)

Remaining inside the context set



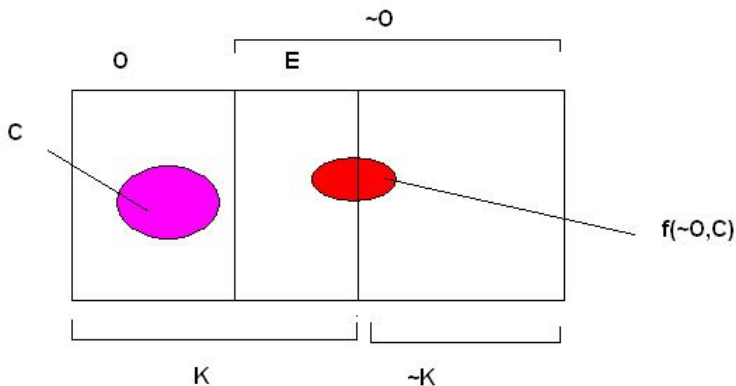
Subjunctive mood

- ▶ *I take it that the subjunctive mood in English and some other languages is a conventional device for indicating that presuppositions are being suspended, which means in the case of subjunctive conditional statements that the selection function is one that **may reach outside the context set** (Stalnaker 1975)*
- ▶ **subjunctive mood**: possibly $f(\phi, C) \not\subseteq C$

Subjunctive Oswald

- (18) If Oswald had not killed Kennedy, someone else would have.
- ▶ Assume $C \subseteq O$: it is assumed Oswald killed Kennedy
 - ▶ Then: necessarily, $f(\neg O, C) \subseteq \neg O$, so $f(\neg O, C) \subseteq \neg C$
 - ▶ Conclusion: for a **counterfactual**, the selection constraint is necessarily violated.
 - ▶ It can be that $f(\neg O, w) \notin K$: the closest-world in which Oswald did not kill Kennedy is not a world in which Kennedy was killed.

Reaching outside the context set



Modus tollens

(19) The murderer used an ice pick. But if the butler had done it, he wouldn't have used an ice pick. So the butler did not do it.

(20) $I, (B > \neg I) \therefore \neg B$

- ▶ “the butler did not do it”: **cannot be a presupposition of the antecedent**. Otherwise, the conclusion would be redundant.

- ▶ $C \subseteq I$
- ▶ but $f(B, w) \models \neg I$
- ▶ hence $f(B, C) \not\subseteq C$.

Moreover:

- ▶ Suppose: $w \in B$, then $f(B, w) = w$, and $w \models \neg I$.
- ▶ So: $w \notin B$.

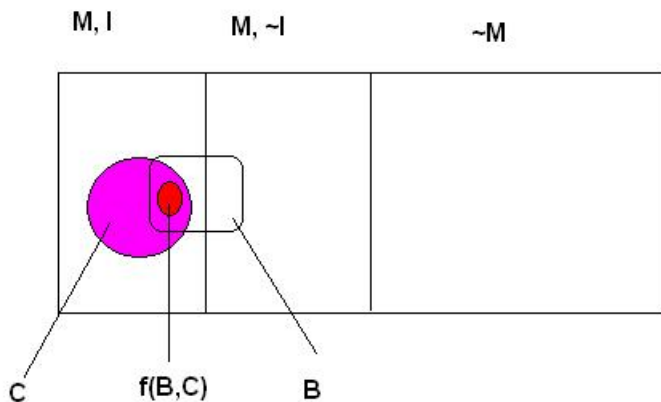
Oddities

Stalnaker's example

- (21) The murderer used an ice pick. # But if the butler did it, he did not use an ice-pick. So the butler did not do it.

Origin of the oddity: "the argument is self-contradictory. the conditional presupposes that there are in fact worlds where the butler did it, there are then claimed to be worlds where no ice-pick was used, contrary to the first premise" (Fintel)

Indicative Modus Tollens



Oddities, cont.

If I did it...

Provocative oddity of O.J. Simpson's book title:

(22) "If I did it, here is how it happened"

Presupposition: maybe I did it. Yet Simpson denies his culpability. Only charitable way out: "I don't remember anything". But then: how can he tell how it actually happened!?

NB. The subjunctive version is no better for a book title, but more appropriate to prove one's innocence in court:

(23) If I had done it, here is how it would have happened.

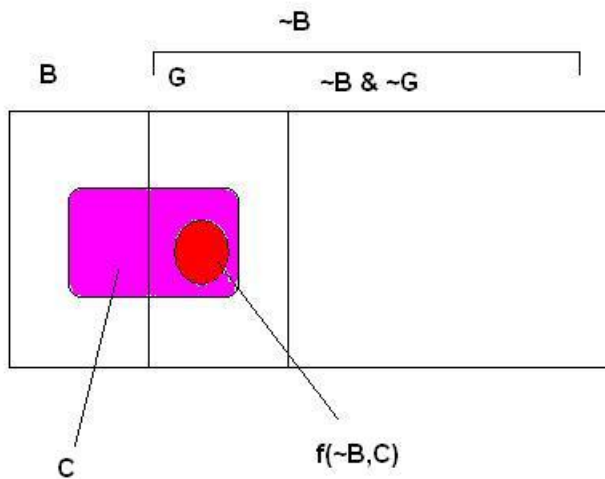
Disjunctive Syllogism

- (24) a. Either the butler or the gardener did it.
b. If the butler did not do it, the gardener did it.

- ▶ Remember: $B \vee G \not\models \neg B \supset G$
- ▶ Stalnaker: the inference is not semantically valid, but it is **pragmatically reasonable**.

Appropriate disjunction

- ▶ **Stalnaker's assumption:** $A \vee B$ is an appropriate utterance with respect to the context set C if C allows each disjunct to be true without the other (ie for every $w \in C$, $w \models \diamond(A \neg B) \wedge \diamond(B \neg A)$):
- ▶ $C \subseteq (B \cup G)$ (after assertion)
- ▶ $C \cap B\bar{G} \neq \emptyset$, $C \cap G\bar{B} \neq \emptyset$ (Stalnaker's assumption)
- ▶ By the selection constraint: $f(\bar{B}, C) \subseteq C$
- ▶ $f(\neg B, C) \subseteq \bar{B}$ (cl 1)
- ▶ hence $f(\neg B, C) \subseteq G$



Interim summary

2 main presuppositions of indicative conditionals:

- ▶ **epistemic possibility of the antecedent:** $C \cap A \neq \emptyset$
- ▶ **context set inclusion:** the antecedent-worlds relative to the context set are part of the context set: $f(A, C) \subseteq C$
So for indicative conditionals one can infer:
 - ▶ $f(A, C) \subseteq A \cap C \neq \emptyset$

An objection by Edgington

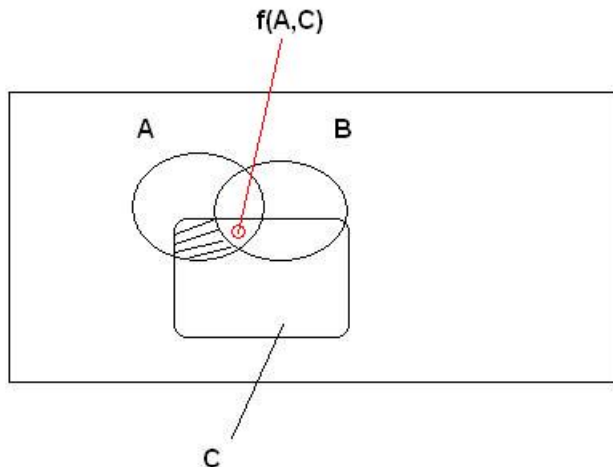
- ▶ Yesterday: we did not have the time to cover no truth value theories
- ▶ According to Edgington's version of this theory: conditionals have **acceptability** conditions, no truth conditions proper.
- ▶ Edgington accepts Adams' thesis
- ▶ She claims that on at least one case, the theory fares better than Stalnaker's

Edgington, cont.

- ▶ Suppose I consider both A and B possible, and am uncertain about both ($P(A) > 0$, $P(B) > 0$, $P(AB) > 0$)
- ▶ I learn that $A \wedge \neg B$ is not the case
- ▶ Then: $P(B|A) = 1$, and by Adams' thesis: I should immediately accept the conditional $A \Rightarrow B$.
- ▶ Not so for Stalnaker:
 - either $w \models A$, then $w \models B$, and $f(A, w) \models B$
 - or $w \models \neg A$. But then one can have: $f(A, w) \models B$, or $f(A, w) \models \neg B$: ie the conditional does not follow.

Stalnaker's answer

- ▶ The problem is solved assuming the selection constraint + the constraint on disjunction + $C \cap (AB) \neq \emptyset$.



Tense and Mood

Overview

- ▶ A growing literature on the topic
- ▶ Ippolito (2002), Schlenker 2005, Arregui (2006), Asher & McCready (2007), Schultz (2007),...
- ▶ Here: we shall only discuss Iatridou's theory: direct connection to Stalnaker's account.

Past morphology

- ▶ An attempt to connect verbal morphology to Stalnaker's ideas
 - (25) If Mary **was** rich, she **would** be happy.
 - (26) If Mary **had** been rich, she **would** have been happy.
- ▶ **Main idea**: counterfactual conditionals make a **non-temporal use** of past morphology

Temporal use of the Past

- ▶ Topic time: $T(t)$ = the time interval we are talking about
- ▶ Utterance time: $C(t)$ = the time interval of the speaker
- ▶ **The Past as precedence**: $T(t)$ precedence $C(t)$

(27) She walked into the room and saw a table.

Modal use of the past

- ▶ Topic worlds: $T(w)$ =the worlds we are taking about
- ▶ Actual world: $C(w)$ = the world(s) of the speaker
- ▶ **The Past as exclusion**: the topic worlds exclude the actual world [or those of the context set].
- ▶ *“the worlds of the antecedent do not include the actual world”* (Iatridou 2000)

Two values of the past

- (28) If he took that syrup, he must feel better now.
[temporal]
- (29) If he took that syrup, he would feel better now. [modal]
- (30) S'il **a pris** ce sirop, il doit se sentir mieux.
- (31) S'il **prenait** ce sirop, il se sentirait mieux.

"When the temporal coordinates of an eventuality are set with respect to the utterance time, aspectual morphology is real. When the temporal coordinates of an event are not set with respect to the utterance time, morphology is always **Imperfect**."

Exclusion as an implicature

(32) John was in the classroom. In fact he still is.

In the same way in which counterfactuality of subjunctive conditionals can be cancelled, exclusion of the actual world/context set from the antecedent worlds can be cancelled.

Empirical adequacy

- ▶ A nice analysis of French so-called **conditional mood** (\neq subjunctive)

- (33)
- a. Si tu **pouvais** nous rendre visite, tu **aimerais** la ville.
 - b. If you could visit us, you would like the city.

- ▶ "Aimerais" = aime- + -r- + **ais** = ROOT + FUT + **IMP**
- ▶ Same pattern for all persons, singular and plural.

Veltman's update semantics

Tichy's puzzle

Consider a man, call him Jones, who is possessed of the following dispositions as regards wearing a hat. Bad weather induces him to wear a hat. Fine weather, on the other hand, affects him neither way: on fine days he puts his hat on or leaves it on the peg, completely at random. Suppose moreover that actually the weather is bad, so Jones is wearing a hat

- (34) If the weather had been fine, Jones would have been wearing his hat.

Intuitions

- ▶ Intuition: sentence false.
- ▶ Alleged prediction from Stalnaker-Lewis (acc. to Tichy): sentence should be true. In the actual world it is raining and Jones is wearing his hat. So any sunny world in which he is wearing his hat is **closer** than any sunny world in which he is not.

Premise semantics

Lewis 1981

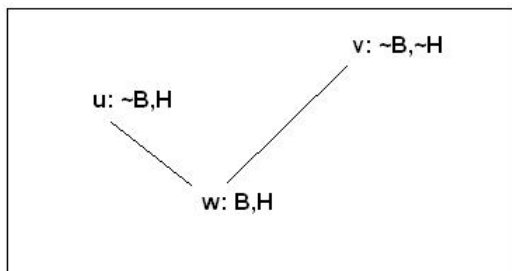
- ▶ In fact the real target of Tichy's point
Simple version of premise semantics:
- ▶ **Premise set:** $P(w)$ set of specific propositions true in w
(remember Kratzer)
- ▶ X is **A-consistent** if $\cap(X \cup \{A\}) \neq \emptyset$
- ▶ X is **A-maximal consistent** if $\neg \exists X'$ s.t. $X \subset X'$ and X' is
A-consistent
- ▶ $\max_A(P(w)) :=$ set of maximal A-consistent sets of $P(w)$.
- ▶ **Semantics:** $w \models_{P(w)} A \Rightarrow C$ iff for all X in $\max_A(P(w))$,
 $\cap(X \cup \{A\}) \subseteq C$

Illustration

Let: H = Jones is wearing a hat; B : the weather is bad.

Suppose $P(w) = \{H, B\}$.

- ▶ $\{H\}$ is the only maximal \bar{B} -consistent subset
- ▶ $H \cap \bar{B} \subseteq H$, ie $w \models_{P(w)} \neg B \Rightarrow H$
- ▶ **Reminder:** $u \leq_w v$ iff for all $X \in P(w)$ such that $w \in X$, $u \in X$.
- ▶ Let $\llbracket H \rrbracket = \{w, u\}$ and $\llbracket B \rrbracket = \{w\}$.
- ▶ Then: $w <_w u <_w v$
- ▶ Hence: $w \models \neg B \square \rightarrow H$



Response

- ▶ When we make the counterfactual assumption that the weather is fine: no reason to maintain the fact that Jones is wearing his hat, since that depends on bad weather in the actual world.

Motivations behind Veltman's semantics

Veltman 2005

- ▶ **Update semantics**: the meaning of a sentence ϕ is an operation on cognitive states S : $S[\phi]$
- ▶ A modified version of simple **premise semantics** (cf. Kratzer)
- ▶ *"making a counterfactual assumption "if it had been the case that ϕ " in state S takes two steps. In the first step any information to the effect that ϕ is in fact false is withdrawn from S , and in the second step the result is updated with the assumption "if it had been the case that ϕ ".*

States

- ▶ Language: $\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi$ and $F := \phi \mid \Box\phi$.
- ▶ $\Box\phi$: "it is a law that ϕ "
- ▶ **Cognitive state**: $S = \langle U_S, F_S \rangle$, where either $\emptyset \neq F_S \subseteq U_S \subseteq W$ or $F_S = U_S = \emptyset$.
- ▶ F_S = facts; U_S = facts + laws

Updates

- ▶ **Update for facts:** $S[\phi] = \langle U_S, F_S \cap \llbracket \phi \rrbracket \rangle$ if $F_S \cap \llbracket \phi \rrbracket \neq \emptyset$, $S[\phi] = \langle \emptyset, \emptyset \rangle$ otherwise.
- ▶ **Update for laws:** $S[\Box\phi] = \langle U_S \cap \llbracket \phi \rrbracket, F_S \cap \llbracket \phi \rrbracket \rangle$ if $F_S \cap \llbracket \phi \rrbracket \neq \emptyset$, $S[\phi] = \langle \emptyset, \emptyset \rangle$ otherwise.

Example

p = the weather is bad; q = Jones is wearing a hat

$$S = W[\Box(p \rightarrow q)][p][q]$$

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
w_4	1	0	0
w_5	1	0	1
w_6	1	1	0
w_7	1	1	1

Situations

- ▶ a **situation** s : a partial world (partial function from atoms to truth-values)
- ▶ s **forces** proposition P within U_S : for all $w \in U_S$ such that $s \subseteq w$, $w \in P$.
- ▶ s is a **basis** for w iff s is a minimal situation such that s forces $\{w\}$ within U_S .

Example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
w_4	1	0	0
w_5	1	0	1
w_6	1	1	0
w_7	1	1	1

Basis for w_6 : $s = \{\langle p, 1 \rangle, \langle r, 0 \rangle\}$

Basis for w_7 : $s = \{\langle p, 1 \rangle, \langle r, 1 \rangle\}$

- ▶ **Basis for w** = minimal set of atomic facts sufficient, given the laws of nature, to select w as a unique world.

Retraction

1. $w \downarrow P = \{s \subseteq w; \text{there is a basis } s' \text{ for } w \text{ such that } s \text{ is a maximal subset of } s' \text{ not forcing } P\}$.
2. $S \downarrow P = \langle U_{S \downarrow P}, F_{S \downarrow P} \rangle$ with:
 - (i) $U_{S \downarrow P} = U_S$
 - (ii) $F_{S \downarrow P} = \{w \in U_S; \text{there is } w' \in F_S \text{ and } s \in w' \downarrow P \text{ such that } s \subseteq w\}$.

Example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
w_4	1	0	0
w_5	1	0	1
w_6	1	1	0
w_7	1	1	1

Basis for w_6 : $s = \{\langle p, 1 \rangle, \langle r, 0 \rangle\}$

Basis for w_7 : $s = \{\langle p, 1 \rangle, \langle r, 1 \rangle\}$

$w_6 \downarrow \llbracket p \rrbracket = \{\langle r, 0 \rangle\}$

$w_7 \downarrow \llbracket p \rrbracket = \{\langle r, 1 \rangle\}$

Counterfactual assumptions

- ▶ $S[\text{if it had been the case that } \phi] = (S \downarrow [\neg\phi])[\phi]$
- ▶ $S \models [\text{if had been } \phi, \text{ would have been } \psi]$ iff $S[\text{if had been } \phi] \models \psi$
- ▶ **Satisfaction:** $S \models \psi$ iff $S[\psi] = S$.

Tichy's example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
w_4	1	0	0
w_5	1	0	1
w_6	1	1	0
w_7	1	1	1

$$S = W[\Box(p \rightarrow q)][p][q]$$

$$w_6 \downarrow \llbracket p \rrbracket = \{\langle r, 0 \rangle\}$$

$$w_7 \downarrow \llbracket p \rrbracket = \{\langle r, 1 \rangle\}$$

$S \downarrow \llbracket p \rrbracket = \langle U_S, U_S \rangle$, for every member of U_S extends $\{\langle r, 0 \rangle\}$ and $\{\langle r, 1 \rangle\}$.

Solution to Tichy's puzzle

Clearly: $(S \downarrow \llbracket p \rrbracket)[\neg p][q] \neq (S \downarrow \llbracket p \rrbracket)[\neg p]$

- (35) It is not true that if the weather had been fine, Jones would have been wearing a hat.

Summary on Veltman

- ▶ Veltman's semantics allows us to refine the basic premise semantics
- ▶ Counterfactuals assumptions work in two steps
- ▶ Difference with indicatives: "making a counterfactual assumption does not boil down to a minimal belief revision" (Veltman 2005)

Summary for today

- ▶ Indicative implies non-counterfactuality, not the other way around
- ▶ indicative and subjunctive conditionals are most likely not so different: but different presuppositions (Stalnaker), different ways of making assumptions (Veltman)

General perspective

Problems for further exploration

- ▶ the probability of conditionals: more work needs to be done!
- ▶ Lewis-Kratzer thesis: better understanding of interaction between conditionals and modals is called for
- ▶ preserving good validities without getting the bad ones back (SDA, IE)
- ▶ tense and mood in conditionals

The End

THANK YOU!